

### Some Brief Thoughts on $17 \times 17$

We know that there is a rectangle free set of  $17 \times 17$  of size 74. Try to prove that there is not rectangle free set of  $17 \times 17$  of size 75. If this is done then the possibilities for how much of each color will be cut down.

Personally I think that each column has 5 of one color and 4 of the other three. Assume this is correct. Then map each column to the color that appears 5 times. Since there are 17 colors there will be some color that appears in 5 times in 5 columns. Is there a rectangle free subset of  $5 \times 17$  which has 5 elements in each column? There is (too bad— if there wasn't that might help in a proof that there was no 4-coloring of  $17 \times 17$ ). However, there is only one (up to perms of columns and rows). Here it is:

(NOTE: THE ABOVE STATEMENT SEEMS TO BE WRONG- THERE SEEM TO BE MORE THAN ONE. SEE A COMMENT ON MY MARCH 17, 2011 POST BY MARZIO DE BIASI.)

	1	2	3	4	5
1	<i>R</i>	<i>R</i>	<i>R</i>		
2	<i>R</i>			<i>R</i>	
3	<i>R</i>				<i>R</i>
4	<i>R</i>				
5	<i>R</i>				
6		<i>R</i>		<i>R</i>	
7		<i>R</i>			<i>R</i>
8		<i>R</i>			
9		<i>R</i>			
10			<i>R</i>	<i>R</i>	
11			<i>R</i>		<i>R</i>
12			<i>R</i>		
13			<i>R</i>		
14				<i>R</i>	
15				<i>R</i>	
16					<i>R</i>
17					<i>R</i>

So, can this be extended to a 4-coloring of  $5 \times 17$  where each color aside from *R* appears 4 times in each column. If you can, that might be a building block in a full 4-coloring. If not that might be the first step in a proof that you cannot 4-color  $17 \times 17$ .

(Added later) ALAS- you CAN 4-color 5x17 where each color blah blah.  
Here it is:

44444333322221111  
43333444421112221  
43222311144443321  
34321432143214412  
33421243214131244