Regular Languages are equivalent to Left-Linear Grammar Languages: An Exposition

William Gasarch *

Univ. of MD at College Park

1 Introduction

This is an exposition of the well known theorem of Chomsky and Miller [1] a language is regular iff the language is generated by a left linear grammar.

Def 1.1 A context free grammar is a *Left Linear Grammar* if every production is either of the form $X \rightarrow \sigma Y$ or $X \rightarrow \sigma$ where $\sigma \in \Sigma \cup \{e\}$.

Note 1.2 Right Linear Grammars can be defined similarly. Everything we say about Left Linear Grammars is true for Right Linear Grammars as well.

2 The Main Theorem

Theorem 2.1 *If M is a DFA then there exists a left linear grammar G such that $L(G) = L(M)$.*

Proof:

Let $M = (Q, \Sigma, \delta, S, F)$ be a DFA.

Let $G = (N, \Sigma, S, P)$ be the left linear grammar defined as follows:

*University of Maryland, College Park, MD 20742, gasarch@cs.umd.edu

1. $N = Q$, the set of states of the original DFA.

2. Σ is the same Σ

3. For every transition $\delta(P, \sigma) = Q$ we have the production $P \to \sigma Q$.

4. For every final state $Q \in F$ we have the production $Q \to e$ where e is the empty string.

5. The start symbol is S, the start state.

We leave the proof that this works to the reader.

∎

Theorem 2.2 *If G is a left linear grammar then there exists a DFA M such that $L(M) = L(G)$.*

Proof:

Let $G = (N, \Sigma, S, P)$ be a left linear grammar.

Let $M = (Q, \Sigma, \delta, S, F)$ be the NDFA defined as follows:

1. $Q = N$.

2. Σ is Σ.

3. For every production $P \to \sigma Q$ we have the transition $\delta(P, \sigma) = Q$.

4. S is S.

5. F is the set of all productions X such that $X \to e$.

∎

The following follows easily from the two theorems above.

Theorem 2.3 *A language is regular iff there is a left linear grammar that generates it.*

References

[1] N. Chomsky and G. Miller. Finite state languages. *Information and Computation*, 1:91–112, 1958.