Review of
**The Space and Motion of Communicating Agents** [1]
**Author: Robin Milner**
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*The reviewer recently learned of Robin Milner's passing away on 20th March 2010. Milner leaves behind a great legacy in theoretical computer science, and was one of the founding fathers of process algebra and the study of concurrency. This review is a summary of his last published book, which describes in detail a mathematical model of reactive systems, namely the formalism of bigraphs.*

## 1 Overview

Milner develops in this book an approach to modelling ubiquitous systems; he begins by observing that modern computing, or rather *informatics*, is more about communication than it is about calculation. He emphasises that, as computing devices increasingly pervade our lives, we will need means of understanding how they interoperate. Even though we may understand thoroughly the functionality of any one device or component, we need to be able to reason about the ways in which that device or component interacts with others, and importantly how an entire network of devices can meet high-level goals and requirements including, among several others, security and privacy constraints.

An analogy is made in the book between properties of ubiquitous systems and the tonal qualities possessed by an orchestra; like a complex system of agents acting and interacting to achieve some goal, the instruments in an orchestra combine in various subtle ways to produce the overall sound. There are qualities of the overall sound which may be translated, or reduced, to qualities of individual instruments, or subgroups of instruments. A ubiquitous system comprises thousands of components, sensors, agents, all of which operate in unison, so the analogy to an orchestra works but there is a difference of scale.

While methods for understanding complex systems exist in the natural sciences, it is still a challenge to develop sufficiently general informatic modelling techniques. Such techniques will be essential in order to design and analyse the information systems of tomorrow, and to this end Milner proposes the *bigraph model*. The purpose of the model is to express clearly, on one hand, the *structure* of ubiquitous systems. This is very significant as there are likely to be several common structures, or system architectures, that will emerge in practical applications; it will be useful to have means of identifying these common structures and extracting key properties. Furthermore, ubiquitous systems are *self-organising*, in that they change their own structure; this poses various design and implementation challenges for humans, who will need ways of visualising and reasoning about such changes.

The two key aspects that needed to be accounted for in order describe these systems are *locality* and *connectivity*, or as Milner prefers, **placing** and **linking**. A formalism suited to the description of ubiquitous systems, he argues, must account for these concepts. The dynamics of a system correspond to *changes in structure and in the links between structures.*

The bigraph model generalises existing process algebraic models such as CCS and the $\pi$-calculus, but requires very rigorous mathematics: the bigraph model described in the book is expressed using a good deal of category theory [6, 7].

## 2 Summary of Contents

The book is divided into three parts dealing, respectively, with the structure, dynamics, and further extensions of bigraphs.

A bigraph is defined as a mathematical object comprising two structures over a set of vertices $V$ and a set of edges $E$, in particular:

- a **place graph**, namely a forest $f$, or set of rooted trees, whose vertices are elements of $V$; these trees embody the nesting of vertices in the bigraph.
- a **link graph**, namely a hypergraph $h$ with vertices $V$ and edges $E$, which simply links together the vertices.

What is essential in the concept of a bigraph is the separation of structure and links. By structure we refer in particular to the nesting of vertices: any vertex in a bigraph can contain any number of other vertices. Links occur between arbitrary vertices, and so there are links that occur on many different levels.

### Part I – Space

The mathematical notion of a bigraph is developed thoroughly in the first of the book, which contains a plethora of formal definitions, notations, and operations on bigraphs. For instance, composition of bigraphs is defined, and a formal semantics of bigraphs as s-categories is given.

An s-category is a form of *precategory*, a structure similar to a category with the difference that composition of arrows may not always be defined. The place graph and link graph of a bigraph are both essentially s-categories. It is also possible to ignore the details of a particular bigraph and study its structure in the abstract, in which case the resulting object is a symmetric partial monoidal (spm) category.

We will not dwell on these aspects here, but it should be noted that the categorical formulation of bigraphs is useful in order to define a graphical calculus that expresses practical operations. Thinking back to applications of the formalism, consider a ubiquitous system with many components: if this system belongs to enterprise $A$, and it needs to be interfaced with another system in enterprise $B$, how can one link the two correctly, unambiguously, and prove that no link has been ignored or incorrectly matched? Interfacing corresponds to composition of bigraphs; composition of maps or arrows is at the core of all category theory, which provides a solid setting in which to study these questions. In essence, the first part of the book develops rigorously the algebra and axioms of bigraphs.

It is worth noting that, in the sixth and final chapter of Part I, the author develops a formal translation of the operators in the process algebra CCS to bigraphs, and similarly for a class of Petri nets, which are widely used in systems modelling. These translations involve mapping to special forms of bigraphs, which are of course defined as needed (e.g. an *ion* is a bigraph consisting of a single vertex with $n$ ports; *ports* are where links to other vertices can be made).

A remark about the graphical notation for bigraphs is in order here. The symbols used in bigraphs are simply a matter of convention; in most examples, the book uses nested ellipses for vertices, and curves for links. However, there are examples which are more intuitive in which vertices can have different shapes to represent aspects of a concrete system, such as rooms, terminals etc. What is important is to be able to represent aspects of real systems, and the author has made sure that bigraphs are flexible enough to cater for many different applications. However, getting the graphics right can be a tricky affair, as evident even on the front cover of the book (which depicts an example bigraph, and has a couple of edges out of kilter).

## Part II – Motion

In the second part of the book, the text develops the dynamics of bigraphs, namely how we can define reactive systems as changing bigraphical structure. Here bigraphs are studied in the abstract, treated as pure mathematical objects without reference to concrete examples as those are simply instances of the general results.

What is key here is how one can define a bigraphical reactive system. A bigraphical reactive system (or simply BRS) is an s-category (such as the place and link graphs of which is a bigraph is made) equipped with *reaction rules*. Reactions express transformations of bigraphs that are meant to correspond to changes and reconfigurations that occur in real systems. In other words, BRSs express *behaviour*.

Given an $s$-category we can define a transition system, similar in nature to the familiar transition systems of CCS or CSP processes. For transition systems corresponding to given BRSs we can define bisimilarity and congruence. These relations allow one to compare given BRSs, again, much in the spirit of classical process algebra.

In order to describe practical systems, it is necessary to impose conditions on the structure of a BRS, otherwise the definition is too broad; for this reason Milner defines in Chapter 8 a set of 'niceness' conditions. Ultimately this leads to a mathematical definition of so-called *prime engaged transitions*, which are the kinds of transitions that are acceptable in practice.

From a practical point of view, understanding the properties of BRSs and corresponding transition systems is important: for any particular domain of application for bigraphs, one needs to formulate a set of reaction rules that express how structures of interest change. These reaction rules need to obey the conditions above mentioned in order to be mathematically sound.

Chapters 9 and 10 develop the theoretical link between BRSs and corresponding transition systems further, with applications to Petri nets and CCS.

## Part III – Development

The final part of the book devotes space to a discussion of issues and possible extensions. The discussion is technical and detailed, and tries to account for certain potential shortcomings and/or issues with bigraphs:

- **tracking:** how to store a history of changes made to a bigraph through successive applications of reaction rules,
- **growth:** how to cope with the expansion of a bigraph as its link and place graphs grow,

- **binding:** how to restrict the scope of a link within a certain space, akin to binding of names in process algebra,

- **stochastics:** how to incorporate relative probabilities or rates in reactions, as particularly relevant in the modelling of e.g. biological systems. The book mentions some existing work on *stochastic bigraphs* [5].

In the final chapter of the book, the author gives a brief historical account of the ideas that led to the development of bigraphs, and links this model to CCS, the $\pi$-calculus, graph and term rewriting methods for systems modelling and analysis. He also cites implementations of algorithms for matching, simulating and inferring from bigraphs, esp. work on bigraphical programming languages at ITU Copenhagen (see `http://www.itu.dk/research/bpl`). Also mentioned are the stochastic bigraphical models of membrane budding discussed in [5].

Milner concludes the main body of the book[2] thus:

> "It can be seen from this work that the bigraph model is being developed through a combination of mathematical intuition and experiment. The experiment involves real interactive systems — both natural, as in biology, and artificial as in ubiquitous computing and business systems. The model tests the hypothesis that the simple ideas of *placing* and *linking*, both physical and metaphorical, unite the mathematical foundation of interactive systems with their applications."

## 3 Opinion

This work is the result of a pioneering effort to formulate, in a mathematically rigorous way, a model of reactive behaviour, as it arises in many applications ranging from computational biology to ubiquitous computing.

The mathematical expression of the model in terms of categories enables several things, namely:

- a solid, unambiguous definition of how and when bigraphs may be composed together (there are similar benefits for the definitions of other operations on these structures),

- a graphical notation which is relatively intuitive and has a formal semantics,

- a unification of existing theories of reactive behaviour, especially process algebras such as CCS and Petri net models.

These benefits must be weighed against the potential criticism that the categorical formulation is abstract and mathematically intensive.

The author himself admits that the definition of bigraphical reactive systems is very broad and that the 'niceness' conditions that are imposed subsequently may be rather taxing. He counters this argument, of course, by pointing out that the generality is needed so as to permit existing models – such as CCS – to be translated to bigraphs.

There is a case to be made for a more usable definition of the transition system corresponding to a BRS, for the labels on transitions are usually identity arrows and do not contain useful information regarding the changes taking place in a particular system. For simulations and analyses

[2] There are a couple of appendices with technical details and solutions to exercises.

of concrete bigraphs, it is likely that further development of this aspect will be desirable. For instance, Michael Goldsmith (University of Warwick) is leading an ONR-funded project that aims to address such issues and ultimately develop practical tool support for bigraphs, especially for use in formal verification [4].

This book should be read by anyone interested in foundations of computer science. Those who are already familiar with CCS and/or $\pi$-calculus will be interested in the development of Milner's thinking about modelling techniques and their applications to various information systems. The generality afforded by the bigraph formalism will interest many, and surely this work will spawn many practical applications and foster the development of simulation and reasoning tools for bigraphs. We are particularly interested in Blackwell's formalism of *spygraphs* [1], which are an extension of bigraphs intended for reasoning about cryptographic protocols. Other relevant work we are aware of is the work of Garner and Hirschowitz [3] and the PORGY project, which is focused on visualisation (see `https://gforge.inria.fr/projects/porgy/`).

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