Statement of Poly VDW Theorem for my Blog
An Exposition by William Gasarch

I first state Van Der Warden’s theorem:

**Notation 0.1** If $W$ is a number then $[W] = \{1, \ldots, W\}$.

**Theorem 0.2** For all $k$ for all $c$ there exists $W = W(k, c)$ such that for all $c$-colorings $\text{COL} : [W] \to [c]$ there exists $a, d$ such that

$$\text{COL}(a) = \text{COL}(a + d) = \cdots = \text{COL}(a + (k - 1)d).$$

What is special about $d, 2d, 3d, \ldots, (k - 1)d$? Can they be replaced by other functions? Yes! They can be replaced by polynomials with zero constant term.

This is the Poly Van Der Warden Theorem:

**Theorem 0.3** For all $p_1, \ldots, p_k \in \mathbb{Z}[x]$ such that $(\forall i)[p_i(0) = 0]$ for all $c$ there exists $W = W(p_1, \ldots, p_k; c)$ such that for all $c$-colorings $\text{COL} : [W] \to [c]$ there exists $a, d$ such that

$$\text{COL}(a) = \text{COL}(a + p_1(d)) = \cdots = \text{COL}(a + p_k(d)).$$