Statement of Poly VDW Theorem for my Blog

An Exposition by William Gasarch

I first state Van Der Warden's theorem:

Notation 0.1 If W is a number then $[W] = \{1, \ldots, W\}$.

Theorem 0.2 *For all k for all c there exists $W = W(k, c)$ such that for all c-colorings $COL : [W] \rightarrow [c]$ there exists a, d such that*

$$COL(a) = COL(a + d) = \cdots = COL(a + (k - 1)d.$$

What is special about $d, 2d, 3d, \ldots, (k - 1)d$? Can they be replaced by other functions? Yes! They can be replaced by polynomials with zero constant term.

This is the Poly Van Der Warden Theorem:

Theorem 0.3 *For all $p_1, \ldots, p_k \in Z[x]$ such that $(\forall i)[p_i(0) = 0]$ for all c there exists $W = W(p_1, \ldots, p_k; c)$ such that for all c-colorings $COL : [W] \rightarrow [c]$ there exists a, d such that*

$$COL(a) = COL(a + p_1(d)) = \cdots = COL(a + p_k(d).$$