Homework 6, Due WED July 23, 2014
NOTE- THIS HW IS TWO PAGES LONG.

1. (0 points) What is your name? Write it clearly. STAPLE your HW.
2. (30 points) For each of the following NIM game determine exactly when Player I wins. (That is, we want a statement like

Player I wins iff $n \equiv B L A H, B L A H(\bmod B L A H)$.
)
(a) $\operatorname{NIM}(1,2)$
(b) $\operatorname{NIM}(1,3)$
(c) $\operatorname{NIM}(1,4)$
(d) $\operatorname{NIM}(1,5)$
(e) $\operatorname{NIM}(1,6)$
(f) $\operatorname{NIM}(1,7)$
3. (20 points) Use your answer to the last problem to solve the following: Let $a \geq 1$. Then Player I wins $\operatorname{NIM}(1, a)$ iff $n$ BLAH BLAH... (NOTEthere may be cases depending on properties of $a$ - such as if $a$ is prime then the condition is XXX, and if $a$ is NOT prime the then condition is YYY. The XXX, YYY may themselves depend on $a$.)

NOTE- THERE IS A SECOND PAGE!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!
4. (20 points)
(a) Consider the 1-pile NIM game where you can remove a SQUARE number of stones. Find the Grundy Numbers for $n=0, \ldots, 25$.
(b) Consider the 1-pile NIM game where you can remove a PRIME number of stones. Find the Grundy Numbers for $n=0, \ldots, 25$. (NOTE- 1 is NOT a prime.)
(c) Consider the following 2-pile NIM game: From pile ONE you can remove a SQUARE number of stones, from pile TWO you can remove a PRIME number of stones. Position $(a, b)$ means that pile ONE has $a$ and pile TWO has $b$. For each of hte following positions say who wins (This will use the Grundy Numbers you computed in the earlier part of this problem.)
i. $(15,19)$
ii. $(20,22)$
iii. $(15,22)$
iv. $(19,22)$
v. $(10,25)$
5. (20 points) Consider the following 2-pile NIM game: From pile ONE you can remove a SQUARE number of stones, from pile TWO you can remove a CUBE number of stones. Write a program in psuedocode that will, given $n$, determine which player wins the game with $a$ in pile ONE and $b$ in pile TWO for all $a, b$ with $a+b \leq 2 n$. (This DOES NOT use Grundy Numbers.)

