

Homework 8, Due WED July 30, 2014

NOTE- THIS HW IS TWO PAGES LONG.

1. (0 points) What is your name? Write it clearly. STAPLE your HW.
2. (30 points)
 - (a) How many roots does the equation $x^3 \equiv 0 \pmod{n^3}$ have? Express your answer as a function of n . Show us the roots!
 - (b) How many roots does the equation $x^4 \equiv 0 \pmod{n^4}$ have? Express your answer as a function of n . Show us the roots!
 - (c) Let $a \geq 5$. How many roots does the equation $x^a \equiv 0 \pmod{n^a}$ have? Express your answer as a function of a and n . Show us the roots!
3. (30 points) Assume we have an 8 letter alphabet $\{a, b, c, d, e, f, g, h\}$. You take a sample text and find the following frequencies which you think (correctly) are indicative of how often they occur in any text you will need to transmit.

a	1
b	2
c	3
d	4
e	5
f	6
g	7
h	8

- (a) If we code a by 000, b by 001, c by 010, ..., h by 111 then what is the average length of a letter?
- (b) Do the Huffman Coding Algorithm to get an encoding of the letters to strings of 0's and 1's. What is the average length of a letter?
- (c) Come up with a coding that is better than the one in part a but not as good as the one in part b.

4. (40 points) On Venus they use coins that are worth $\{1, 3, 6\}$.

Let a_n be how many ways to make change of n cents using just pennies

Note that, for all n , $a_n = 1$.

Let b_n be how many ways to make change of n cents using pennies and 3-cent coins.

Let c_n be how many ways to make change of n cents using pennies and 6-cent coins and 3-cent coins.

- (a) Write a recurrence for b_n
- (b) Write a formula for b_n .
- (c) Write a recurrence for c_n
- (d) How many ways are there to make change of 30 cents? Show all work.