Homework 8, Due Mon July 27, 2015

WARNING- THE HW IS TWO PAGES.
REMINDER- MIDTERM REDOS DUE MONDAY

1. (20 points) Conrad and Victoria Grayson host a party with 10 people.

   (a) How many ways can the 10 people be ordered? For example, if the people are $A, B, C, D, E, F, G, H, I, J$ then $ABCDEFGHIJ$ is a different from $BCDAEFGHIJ$.

   (b) Assume three of the people are triplets that nobody can tell apart. Then how many ways can they be ordered?

2. (25 points) Alice and Bob are going to to secret sharing with cards and of course Eve is there too! The cards are $\{1, 2, \ldots, 10\}$. Initially Alice has 4 cards, Bob has 4 cards, and 2 cards. They agree that if they agree that Alice has a and Bob has b then if $a < b$ the bit shared is 0, and if $a > b$ then the bit shared is 1. Assume Alice has $\{1, 3, 8, 9\}$ Bob has $\{2, 4, 5, 6\}$, and Eve has $\{7, 10\}$. We abbreviate *Alice says I have one of* a, b by *Alice*—(a, b) Assume the following sequence happens:

   - Alice—(1,4)
   - Alice—(3,5)
   - Alice—(9,6)
   - Alice—(8,2)

   What bit sequence do Alice and Bob share?

3. (25 points) Alice and Bob agree ahead of time on the following ordering of partitions of $\{1, 2, 3, 4\}$ into 2 cards for Alice and 2 cards for Bob

C_1	$\{1, 2\}$	$\{3, 4\}$
C_2	$\{1, 3\}$	$\{2, 4\}$
C_3	$\{1, 4\}$	$\{2, 3\}$
C_4	$\{2, 3\}$	$\{1, 4\}$
C_5	$\{2, 4\}$	$\{1, 3\}$
C_6	$\{3, 4\}$	$\{1, 2\}$

Assume that Alice has $\{1, 3\}$, Bob has $\{2, 4\}$ and Eve has NOTHING. To share secret bits with Bob, Alice says the following:

$$C_2, C_6, C_4, C_1$$

What is the shared secret bits that Alice and Bob now share?

4. (30 points) Alice and Bob are going to to secret sharing with cards and of course Eve is there too! The cards are $\{1, 2, \ldots, 20\}$. Initially Alice has 8 cards, Bob has 8 cards, and 4 cards.

   (a) Give an EXAMPLE of cards for Alice and cards for Bob and a SCENARIO where at the end Alice has 6 and Bob has 6 and Eve has NO cards.

   (b) Alice and Bob must have ahead of time devised a table like the C_i table in the last problem. How many C_i's will the have? More directly— how many ways are there to partition $\{1, 2, 3, 4, 5, 6, 7, 9, 9, 10, 11, 12\}$ into two sets of size 6. (DO NOT do such a table- it will be large! We just want to know how big it is.)

   (c) Assume that eventually Alice has the six cards $\{1, 3, 5, 7, 9, 18\}$ and Bob has the six cards $\{6, 10, 12, 13, 19, 20\}$. OH, but the table only has partitions of $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ into two sets of size 6. What do Alice and Bob do?