Homework 9, Due Tue July 28, 2015

WARNING- THE HW IS TWO PAGES.

1. (25 points)

   (a) Find a trick for the problem of determining if a number is divisible by 11. Do you think it is really a trick?

   (b) Find a trick for the problem of determining if a number is divisible by 13. Do you think it is really a trick?

2. (25 points) Consider the following error detection scheme: The content is 10 digits long: $a_{10}a_9 \cdots a_1$. We also add the NUMBER:

   $a_{10} + \cdots + a_1$

   (NOTE- we do not do a MOD)

   We will assume that this NUMBER is always transmitted correctly.

   EXAMPLE: If you want to transmit $(1, 8, 2, 1, 9, 3, 3, 4, 5, 9)$ you actually transmit $(1, 8, 2, 1, 9, 3, 3, 4, 5, 9, 45)$

   (a) What is the most digits that $a_{10} + \cdots + a_1$ has?

   (b) What is the probability that a single-digit-error will NOT be detected?

   (c) What is the probability that a double-error will not be detected? (that is, two different digits are transmitted incorrectly).

   (d) What is the probability that an adjaceny-transposition error will NOT be detected?

3. (25 points) You want to transmit 18 real bits and 2 check-bits. You wan to catch ALL single-digit errors and ALL adjacency-transposition errors. How would you do this? Be very clear here- the students may have LOTS of different answers and we don't want to drive the TA crazy.

4. (25 points) Assume we have an 8 letter alphabet $\{a, b, c, d, e, f, g, h\}$. You take a sample text and find the following frequencies which you think (correctly) are indicative of how often they occur in any text you will need to transmit.

a	1
b	1
c	3
d	3
e	6
f	6
g	9
h	10

   (a) If we code a by 000, b by 001, c by 010, ..., h by 111 then what is the average length of a letter?

   (b) Do the Huffman Coding Algorithm to get an encoding of the letters to strings of 0's and 1's. What is the average length of a letter?

   (c) Come up with a coding that is better than the one in part a but not as good as the one in part b.