1. (0 points) What is your name? Write it clearly. STAPLE your HW.

2. (20 points)
   
   (a) Let $p$ be a prime. Determine $\phi(p^2)$ (the number of numbers in 
   \{1, \ldots, p^2\} that are rel prime to $p^2$).
   
   (b) Let $p$ be a prime. Let $a \geq 1$. Determine $\phi(p^a)$
   
   (c) Let $a, b$ be relatively prime. Show that $\phi(ab) = \phi(a)\phi(b)$.
   
   (d) Let $p_1, \ldots, p_L$ be primes. Let $a_1, \ldots, a_L \geq 1$. Use the last two 
   items to determine a formula for $\phi(p_1^{a_1}p_2^{a_2}\cdots p_L^{a_L})$.

3. (30 points) For each of the following scenarios determine (1) how many 
   bits Alice and Bob share in the case (the question do you have an $x$ or 
   a $y$? always gets the answer YES, (2) how many bits Alice and Bob 
   share in the case (the question do you have an $x$ or a $y$? always gets 
   the answer NO. Use that the number of bits for $(a, b, 0)$ is $\left\lfloor \log_2 \left( a^n b^n \right) \right\rfloor$.
   
   (a) $a = 10, b = 10, e = 10$.
   (b) $a = 20, b = 10, e = 10$.
   (c) $a = 10, b = 10, e = 20$.
   (d) $a = n, b = n, e = n$ (so both answers are a function of $n$). (Assume 
   $n$ is even. Leave in terms of $n$-choose notation.)
   
   (e) $a = 2n, b = n, e = n$ (so both answers are a function of $n$). 
   (Assume $n$ is even. Leave in terms of $n$-choose notation.)
   
   (f) $a = 2n, b = n, e = 2n$ (so both answers are a function of $n$). 
   (Assume $n$ is even. Leave in terms of $n$-choose notation.)
4. (20 points) $A_1, A_2, A_3, A_4$ are people. Zelda has a secret $s$. She wants to give everyone strings such that the following sets of people (and their supersets) can find the secret, but no other group can:

\{A_1, A_2\}
\{A_1, A_3\}
\{A_1, A_4\}
\{A_2, A_3, A_4\}

5. (30 points). $A_1, A_2, A_3, A_4$ are people. Zelda has a secret $s$. She wants that if any set of two of them get together they can determine the secret but no one person can.

(a) If Zelda uses the prime 7, $A_1$ gets $f(1) = 6$, and $A_2$ gets $f(2) = 5$ then what is the secret?

(b) If Zelda uses the prime 11, $A_1$ gets $f(1) = 6$, and $A_2$ gets $f(2) = 5$ then what is the secret?

(c) If Zelda uses the prime 13, $A_1$ gets $f(1) = 6$, and $A_2$ gets $f(2) = 5$ then what is the secret?