HW 3 HONR 209M. Morally DUE Tuesday Sep 24

SOLUTIONS

1. (0 points) What is your name? Write it clearly. Staple your HW. When is the first midterm? When is the final?

   NOTE- THIS HW IS TWO PAGES, DO NOT MISS SECOND PAGE.

2. (60 points) Alice and Bob are looking at cake that we think of as the interval $[0, 1]$. Alice's valuation is uniform on $[0, \frac{3}{4}]$. Bob's valuation is uniform on $[\frac{1}{8}, 1]$.

   (a) Assume that neither knows the others valuation. Assume they do Cut and Choose with Alice cutting, Bob Choosing. Where is the cut going to be? How much does Alice get? How much does Bob get?

   (b) Assume Alice knows Bob's valuation. Assume they do Cut and Choose with Alice cutting, Bob Choosing. Where should Alice cut to do BETTER than if she didn't know (which is the case in part a)? How much does Alice get? How much does Bob get? (NOTE- there are many answers. Pick one where Alice does BETTER than she does in part (a). Try to make it much better but don't try to optimize it as this is actually not possible.)

   (c) If Alice and Bob reveal their honest valuation to each other and agree to choose a value x to cut at so that they both have the same amount, What x do they choose? How much does Alice get? How much does Bob get? What is the TOTAL of what Alice gets PLUS what Bob gets?

   (d) If they both reveal their honest valuation to each other and agree to choose a value x to cut at so that the SUM of what Alice and Bob gets is maximized, what x do they choose, How much does Alice get? How much does Bob get? What is the TOTAL of what Alice gets PLUS what Bob gets? (If its less than the TOTAL in part c then your answer is wrong.)

SOLUTION TO PROBLEM 2

Note that if Alice gets an interval contained in $[0, \frac{3}{4}]$ of length L, then she gets value $\frac{4L}{3}$. Note that if Bob gets an interval of length L contained in $[\frac{1}{8}, 1]$ then he gets value $\frac{8L}{7}$.

a) Alice cuts at the $\frac{3}{8}$ place. Bob will pick the right part.

Alice gets $[0, \frac{3}{8}]$ so gets $\frac{4}{3} \times \frac{3}{8} = \frac{1}{2}$ (the cutter ALWAYS gets 1/2)

Bob gets $[\frac{3}{8}, 1]$ so gets $\frac{5}{8} \times \frac{8}{7} = \frac{5}{7} \sim 0.714$

b) If Alice knows Bob's valuation she will cut very close to Bob's 1/2 point, but shaded just a bit to the left so that Bob will take the right piece, and Alice gets the Left piece. (Alice wants this since alot of the right piece is not good for her.) Alice should cut at $\frac{9}{16} - \epsilon$ where we pick ϵ later but it will be small.

Bob gets $[\frac{9}{16} - \epsilon, 1]$ so gets $(1 - \frac{9}{16} + \epsilon)\frac{8}{7} = (\frac{7}{16} + \epsilon)\frac{8}{7} = \frac{1}{2} + \frac{8\epsilon}{7}$. NOTE- just a wee bit more than $\frac{1}{2}$.

Alice gets $[0, \frac{9}{16} - \epsilon]$ so gets $(\frac{9}{16} - \epsilon)\frac{4}{3} = \frac{3}{4} - \frac{4\epsilon}{3}$. NOTE- just a wee bit less than 3/4.

c) If the cut is at $x \in [\frac{1}{8}, \frac{3}{4}]$ then

Alice gets $[0, x]$ so gets $\frac{4x}{3}$

Bob gets $[x, 1]$ so gets $(1 - x)\frac{8}{7}$.

$$\begin{aligned} \frac{4x}{3} &= \frac{8(1-x)}{7} \\ \frac{x}{3} &= \frac{2(1-x)}{7} \\ 7x &= 6(1-x) \\ 7x &= 6 - 6x \\ 13x &= 6 \\ x &= \frac{6}{13} \end{aligned}$$

Alice gets $[0, \frac{6}{13}]$ so has $\frac{6}{13}\frac{4}{3} = \frac{8}{13} \sim 0.615$

Bob should get the same, but lets check

Bob gets $[\frac{6}{13}, 1]$ so has $\frac{7}{13} \times \frac{8}{7} = \frac{8}{13}$.

NOTE that the TOTAL of what they get is $\frac{16}{13} \sim 1.23$

d) We want to maximize

$$\frac{4x}{3} + (1-x)\frac{8}{7}$$

on the interval $x \in [\frac{1}{8}, \frac{3}{4}]$.

$$\frac{4x}{3} + (1-x)\frac{8}{7} = \frac{4x}{21} + \frac{8}{7}.$$

Since the coeff of x is positive this is maximized when x is as large as possible which is $x = \frac{3}{4}$.

Alice gets $[0, \frac{3}{4}]$ so she has 1

Bob gets $[\frac{3}{4}, 1]$ so he has $\frac{1}{4} \times \frac{8}{7} = \frac{2}{7}$. (Bob is NOT happy about this.)

NOTE- the TOTAL of what they get is $1 + \frac{2}{7} = \frac{9}{7} \sim 1.28$.

3. (40 points) Assume that Alice and Bob have the following points values for the items PICASSO, CAR, HOUSE, MILLION DOLLARS, ISLAND.

	Alice	Bob
PICASSO	15	40
CAR	20	15
HOUSE	20	15
MILLION	25	30
ISLAND	20	0

Also assume that the MILLION is the only fluid item.

(a) Do the AW procedure. State who gets what and how many points they both get (which should be the same).

(b) Imagine a scenario where Alice has MUCH better lawyers than Bob so the agreement is that Alice will get TWICE as many points as Bob. Find a way to allocate goods so that this happens. (SO in the end, for example, it may be that Alice has 80 using her eval, and Bob has 40 using his eval.) State who gets what and how many points each gets. (Alice's should be twice as much as Bob's.)

SOLUTION TO PROBLEM 3

We need the following for both parts:

	Alice	Bob
PICASSO	15	40*
CAR	20*	15
HOUSE	20*	15
MILLION	25	30*
ISLAND	20*	0
TOTAL	55	70

a) The MILLION is the only FLUID item. So Bob will give some of it to Alice to make them EQUAL

$$\begin{aligned} 55 + 25x &= 70 - 30x \\ 55x &= 15 \\ x &= 3/11 \end{aligned}$$

So 3/11 of the money goes to Alice, and 9/11 to Bob.

With that

Alice has $55 + 25(3/11) = 61.81...$

Bob has the same (check).

b) We now want to have Alice get TWICE as much. So we want

$$\begin{aligned} 55 + 25x &= 2(70 - 30x) \\ 55 + 25x &= 140 - 60x \\ 85x &= 85 \\ x &= 1 \end{aligned}$$

WOW- Bob has to fork over ALL of the Million dollars to Alice! When he does that

Alice has $55 + 25 = 80$.

Bob gets half of that (check).