

HW 4 HONR 209M. Morally DUE Tuesday Oct 1

1. (0 points) What is your name? Write it clearly. Staple your HW. When is the first midterm? When is the final?

NOTE- THIS HW IS TWO PAGES, DO NOT MISS SECOND PAGE.

2. (60 points) Alice and Bob are looking at cake that we think of as the interval $[0, 1]$. Let $f(x) = 2x$ and $g(x) = 1$. Alice's valuation is $v(a, b) = \int_a^b f(x) = b^2 - a^2$. Bob's valuation is $v(a, b) = \int_a^b g(x) = b - a$.
 - (a) Assume that neither knows the others valuation. Assume they do Cut and Choose with Alice cutting, Bob Choosing. Where is the cut going to be? How much does Alice get? How much does Bob get? What is Alice + Bob (the total)?
 - (b) Assume Alice knows Bob's valuation. Assume they do Cut and Choose with Alice cutting, Bob Choosing. Where should Alice cut to do BETTER than if she didn't know (which is the case in part a)? How much does Alice get? How much does Bob get? What is Alice + Bob (the total)? (NOTE- there are many answers. Pick one where Alice does BETTER than she does in part (a). Try to make it much better but don't try to optimize it as this is actually not possible.)
 - (c) If Alice and Bob reveal their honest valuation to each other and agree to choose a value x to cut at so that they both have the same amount, What x do they choose? How much does Alice get? How much does Bob get? How much does Alice get? How much does Bob get? What is Alice + Bob (the total)?
 - (d) If they both reveal their honest valuation to each other and agree to choose a value x to cut at so that the SUM of what Alice and Bob gets is maximized, what x do they choose, How much does Alice get? How much does Bob get? What is the TOTAL of what Alice gets PLUS what Bob gets? (If its less than the TOTAL in part c then your answer is wrong.)

3. (40 points) Alice and Bob want to split a cake in the ratio (121 : 100). (For this problem you can assume that for $(a : 1)$ the number of cuts is $\lceil \lg(a + 1) \rceil$ and for $(1 : b)$ its $\lceil \lg(b + 1) \rceil$. Hence you can stop the tree if either number is 1.)
- (a) Give the protocol, in the form of a tree, for when Alice and Bob do the near-halves protocol (reducing when it is possible). How many cuts does it take in the worst case?
 - (b) Give a protocol where the FIRST step is that Bob cuts the cake in ratio (1 : 220) and either Alice or Bob takes the 1-piece, and after that do the near-halves protocol. How many cuts does it take in the worst case?