HW 5 HONR 209M. Morally DUE Tuesday Oct 8

1. (0 points) What is your name? Write it clearly. Staple your HW. When is the first midterm? When is the final?

   NOTE- THIS HW IS TWO PAGES, DO NOT MISS SECOND PAGE.

2. (60 points) Alice and Bob are looking at cake that we think of as the interval $[0,1]$. Let $f(x) = 2x$ and $g(x) = x + \frac{1}{2}$. Alice's valuation is $v_A(a,b) = \int_a^b f(x)dx = b^2 - a^2$. Bob's valuation is $v_B(a,b) = \int_a^b g(x)dx = \frac{b^2-a^2}{2} + \frac{b-a}{2}$

   (a) Find the number x_{LA} such that if the cut is at x_{LA} and Alice takes the interval $[0, x_{LA}]$ then Alice gets exactly $\frac{1}{3}$. Note that if $x \geq x_{LA}$ and Alice takes $[0,x]$ then she has $\geq \frac{1}{3}$. (We call it x_{LA} since Left piece is going to Alice.)

   (b) Find the number x_{RA} such that if the cut is at x_{RA} and Alice takes the interval $[x_{RA}, 1]$ then Alice gets exactly $\frac{1}{3}$. Note that if $x \leq x_{RA}$ and Alice takes $[x,1]$ then she has $\geq \frac{1}{3}$. (We call it x_{RA} since Right piece is going to Alice.)

   (c) Find the number x_{LB} such that if the cut is at x_{LB} and Bob takes the interval $[0, x_{LB}]$ then Bob gets exactly $\frac{2}{3}$. Note that if $x \geq x_{LB}$ and Bob takes $[0,x]$ then he has $\geq \frac{2}{3}$. (We call it x_{LB} since Left piece is going to Bob.)

   (d) Find the number x_{RB} such that if the cut is at x_{RB} and Bob takes the interval $[x_{RB}, 1]$ then Bob gets exactly $\frac{2}{3}$. Note that if $x \leq x_{RB}$ and Bob takes $[x,1]$ then he has $\geq \frac{2}{3}$. (We call it x_{RB} since Right piece is going to Bob.)

   (e) Find the set of ALL x such that if the cut is at x and Alice takes the left and Bob takes the right, Alice gets $\geq \frac{1}{3}$ and Bob get $\geq \frac{2}{3}$.

   (f) Find the set of ALL x such that if the cut is at x and Alice takes the right and Bob takes the left, Alice gets $\geq \frac{1}{3}$ and Bob get $\geq \frac{2}{3}$.

3. (40 points) Alice, Bob, Carol, and Donna want to split cake in ratio $(a : b : c : d)$. Give a protocol for this. (HINT: The first step is to use the Alice-Bob-Ratio $(a : b : c)$ protocol.)