HW 6 HONR 209M. Morally DUE Tuesday Oct 15

SOLUTIONS

- 1. (0 points) What is your name? Write it clearly. Staple your HW. When is the first midterm? When is the final?
- 2. (60 points)

Alice's valuation is uniform on [0, 1]Bob's valuation is uniform on [1/3, 2/3]Carol's valuation is uniform on [1/4, 7/8].

- (a) Write down all the needed equations and inequalities for a Linear program whose solution is a proportional division which maximizes total happiness. Say what each variable stands for and what each equation is enforcing. (NOT required to solve, but you can using an LP solver.)
- (b) Write down all the needed equations and inequalities for a Linear program whose solution is a proportional division which minimizes individual unhappines Say what each variable stands for and what each equation is enforcing. (NOT required to solve, but you can using an LP solver.)
- (c) Write down all the needed equations and inequalities for a Linear program whose solution is an envy free division which maximizes total happiness. Say what each variable stands for and what each equation is enforcing. (NOT required to solve, but you can using an LP solver.)
- (d) Assume that Alice KNOWS Carol's valuation. Write down AN equation that means Alice (using Alices valuation) gets more than Carol (using Carol's valuation).

SOLUTION TO PROBLEM 2

a) The intervals are

 $I_1 = [0, 1/4]$ – Alice gets all of this.

 $I_2 = [1/4, 1/3]$ –Alice and Carol get this $I_3 = [1/3, 2/3]$ -ALL get this $I_4 = [2/3, 7/8]$ -Alice and Carol get this $I_5 = [7/8, 1]$ - Alice gets all of this. x_{Pi} is how much of interval I_i Person P gets. ALREADY SET: $x_{A1} = 1/4, x_{B1} = x_{C1} = 0$ $x_{B2} = 0$ $x_{B4} = 0.$ $x_{A5} = 1/8$. $x_{B5} = x_{C5} = 0$. EQUATIONS SO THAT THE VARS MAKE SENSE: $x_{A2} + x_{C2} = 1/3 - 1/4 = 1/12$ $x_{A3} + x_{B3} + x_{C3} = 2/3 - 1/3 = 1/3.$ $x_{A4} + x_{C4} = 7/8 - 2/3 = 5/24.$ For all $i, P x_{Pi} \ge 0$. EQUATIONS SO THAT THEY ALL GET AT LEAST 1/3: NOTE: Alice's mult is 1, Bob's mult is 3, Carol's mult is 8/5Alice: $1/4 + x_{A2} + x_{A3} + x_{A4} + 1/8 \ge 1/3$. (ALREADY SATISFIED!) Bob: $3(x_{B3}) \ge 1/3$ Carol: $(8/5)(x_{C2} + x_{C3} + x_{C4}) \ge 1/3$. MAX TOTAL HAPPY: MAX the sum of what I have above for Alice, Bob, Carol. b) To minimize unhappiness we have a new var t and do the following replacements: Replace the EQUATIONS SO THAT THEY ALL GET AT LEAST 1/3 with

Alice: $1/4 + x_{A2} + x_{A3} + x_{A4} + 1/8 \ge t$ Bob: $3(x_{B3}) \ge t$ Carol: $(8/5)(x_{C2} + x_{C3} + x_{C4}) \ge t$ Replace what you maximize by t. c) Envy Free! ADD the following 6 equations: Alice not envious of Bob:

$$x_{A1} + x_{A2} + x_{A3} + x_{A4} + x_{A5} \ge x_{B1} + x_{B2} + x_{B3} + x_{B4} + x_{B5}.$$

Bob not enviious of Alice:

$$x_{B3} \ge x_{A3}$$
.

Alice not env of Carol:

$$x_{A1} + x_{A2} + x_{A3} + x_{A4} + x_{A5} \ge x_{C1} + x_{C2} + x_{C3} + x_{C4} + x_{C5}.$$

Carol not env of Alice:

$$x_{C2} + x_{C3} + x_{C4} \ge x_{A2} + x_{A3} + x_{A4}.$$

Bob not env of Carol:

$$x_{B3} \ge x_{C3}.$$

Carol not env of Bob:

$$x_{C2} + x_{C3} + x_{C4} \ge x_{B2} + x_{B3} + x_{B4}.$$

d)

$$1/4 + x_{A2} + x_{A3} + x_{A4} + 1/8 \ge (8/5)(x_{C2} + x_{C3} + x_{C4})$$

3. (40 points)

- (a) Present a protocol that does 3-person proportional division with only 3 cuts (Hint: its really Divide and Conquer).
- (b) Find out EXACTLY how many cuts Divide and Conquer takes for n = 4, 5, 6, 7, 8 (Hint: Read the notes on Divide and Conquer. Gee— you should do that in any case.)
- a) (Some said DO TRIM. Thats fine, but here is what I had in mind.)
- (a) Alice and Bob cut it (1/3-2/3)
- (b) Assume Alice has the left-most cut. KEY: Alice is happy taking the L or splitting R with a person. Bob is happy splitting R with a person only.
- (c) Carol says either L IS $\geq 1/3$ or R is $\geq 2/3$.
 - i. If Carol says L is $\geq 1/3$ then Carol gets L and Bob and Alice split R
 - ii. If Carol says R is $\geq 2/3$ then Alice gets L and Bob and Carol split R
- b)

n = 4: All but one makes a cut, so thats 3 cuts. Then its in two pieces L and R and there will be 2 on each. So

$$T(4) = 3 + T(2) + T(2) = 3 + 1 + 1 = 5.$$

n = 5: All but one make a cut, so thats 4 cuts. Then its two pieces, one of which will be split among 3 people, and the other amoung 2 T(5) = 4 + T(3) + T(2) = 4 + 3 + 1 = 8

n = 6: All but one make a cut, so thats 5 cuts. Then its two pieces, one of which will be split among 3 people, and the other amoung 3 T(6) = 5 + T(3) + T(3) = 5 + 3 + 3 = 11

n = 7: All but one make a cut, so thats 6 cuts. Then its two pieces, one of which will be split among 3 people, and the other amoung 4 T(7) = 6 + T(4) + T(3) = 6 + 5 + 3 = 14

n = 8: All but one make a cut, so thats 7 cuts. Then its two pieces, one of which will be split among 4 people, and the other amoung 4 T(8) = 7 + T(4) + T(4) = 7 + 5 + 5 = 17