

HW 7 HONR 209M. Morally DUE Tuesday Oct 29
THERE IS ANOTHER PROBLEM ON PAGE 2.

SOLUTIONS

1. (0 points) What is your name? Write it clearly. Staple your HW. When is the first midterm? When is the final?

2. (50 points) For the 3-person ENVY free protocol do the following.
 - (a) Show that if Alice does NOT cut all the pieces equally then she might feel envy.
 - (b) Show that if Alice DOES cut all the pieces equally then she will not feel envy, regardless of what the others do.
 - (c) Show that if Bob thinks the top two pieces are not equal but DOES NOT trim to a tie then he might feel envy.
 - (d) Show that if Bob thinks the top two pieces are equal by trims anyway he might feel envy.
 - (e) Show that if Bob DOES trim honestly then he cannot feel envy, regardless of what the others do.
 - (f) Assume that Bob got the trimmed piece and that, in phase 2, Carol divides the Trim. Show that if Carol does NOT cut the Trim equally then she might feel envy.

EVERYONE GOT THIS RIGHT! SO NO SOLUTION NEEDED!
THERE IS ANOTHER PROBLEM ON PAGE 2.

3. (50 points) ASSUME the following:

- There IS a 2-cut $(3, 4y, y)$ protocol
- There IS a 3-cut $(4, 6y, y)$ protocol
- There IS a 4-cut $(5, 8y, y)$ protocol

Show that there IS a 5-cut $(6, 10y, y)$ protocol. HINT: Begin as follows:

Alice cuts a piece (equally) so now there are pieces P_1, P_2 . B,C,D,E,F all decide which of P_1 or P_2 they prefer. Assume they all like P_1 (the other case is easy but may be on the midterm).

Let $B(2)$ be how much B likes P_2 . Similar for $C(2)$, etc.

Ask them for their values and then form the ordering which we will assume is

$$B(2) \leq C(2) \leq D(2) \leq E(2) \leq F(2)$$

ASK the questions:

$$F(2) \geq 2y?$$

(NOTE: If NO then

$$B(2) \leq C(2) \leq D(2) \leq E(2) \leq F(2) < 2y$$

)

$$E(2) \geq 4y?$$

(NOTE: If NO then

$$B(2) \leq C(2) \leq D(2) \leq E(2) < 4y$$

)

YOU figure out what to do with the answers.

SOLUTION TO PROBLEM 3.

Note that A thinks P_2 is worth $5y$.

Case 1: If the first question is a NO then

$$B(2) \leq C(2) \leq D(2) \leq E(2) \leq F(2) < 2y$$

SO B, C, D, E, F all think $P_2 < 2y$. So they all think $P_1 \geq 8y$. A gets P_2 , B, C, D, E, F (5 people) split P_1 using $(5, 8y, y)$.

Case 2: If the first question is YES but the second question is NO then

$$F(2) \geq 2y$$

So A and F split P_2 .

But the NO to the second question means

$$B(2) \leq C(2) \leq D(2) \leq E(2) < 4y$$

So B, C, D, E all think $P_2 < 4y$ so $P_1 \geq 6y$. These FOUR people use $(4, 6y, y)$ protocol.

Case 3: Answers are YES, YES. Then

$$F(2) \geq 2y$$

and

$$E(2) \geq 4y$$

But

$$4y \leq E(2) \leq F(2)$$

So E, F both think P_2 is at least $4y$. A thinks its $5y \geq 4y$. So A, E, F use $(3, 4y, y)$ protocol.

B, C, D all think $P_1 \geq 5y$ (its the bigger half) so they all use $(3, 4y, y)$ protocol.