HW 9 HONR 209M. Morally DUE Tuesday Nov 26, but will take TUESDAY Dec 3

SOLUTIONS

1. (0 points) What is your name? Write it clearly. Staple your HW. When is the final?

2. (50 points) Auctions: In class we did that if there was a TIE then there is NO SALE. For this problem in an action if there is a TIE then the people who tied split the cost and split the item. (If tie with ONE other person, both pay 1/2 and get 1/2. If tie with TWO other people, all three pay 1/3 and get 1/3.) Throughout this problem Alice thinks the bids are in \( \{0, \ldots, b-1\} \) and Alice thinks the item is worth \( v \).

(a) If there is only one other bidder than what should Alice bid to maximize her profit? Show your work.

(b) (EXTRA CREDIT) If there are two other bidder than what should Alice bid to maximize her profit? Show your work.

(c) (EXTRA CREDIT) If there are \( n \) other bidder than what should Alice bid to maximize her profit? Show your work.

SOLUTION TO PROBLEM 2

a) Assume Alice bids \( x \). Only one other bidder.

Prob of Alice WINNING is \( \frac{x}{b} \). Prob of Alice TIEING is \( \frac{1}{b} \). Hence Alice’s expected profit is

\[
\frac{x}{b}(v-x) + \frac{1}{b} \left( \frac{v}{2} - \frac{x}{2} \right) = \frac{1}{b} \left( x(v-x) + \frac{v-x}{2} = \frac{1}{b}(v-x)(x + \frac{1}{2}) \right).
\]

We ignore the \( \frac{1}{b} \) term and take the derivative using the product rule.

\[
f'(x) = (v-x) - (x + \frac{1}{2}) = v + \frac{1}{2} - 2x
\]

Set this to 0 to get \( x = \frac{v+0.5}{2} \).

3. (50 points) Redo the PIRATES problem with the following change: The CAPTAIN gets two votes. Figure out the problem for \( n = 1, 2, 3, \ldots, 10. \)

SOLUTION OMITTED