

HW 9 HONR 209M. Morally DUE Tuesday Nov 26, but will take TUESDAY Dec 3

SOLUTIONS

1. (0 points) What is your name? Write it clearly. Staple your HW. When is the final?
2. (50 points) Auctions: In class we did that if there was a TIE then there is NO SALE. For this problem in an action if there is a TIE then the people who tied split the cost and split the item. (If tie with ONE other person, both pay $1/2$ and get $1/2$. If tie with TWO other people, all three pay $1/3$ and get $1/3$.) Throughout this problem Alice thinks the bids are in $\{0, \dots, b-1\}$ and Alice thinks the item is worth v .
 - (a) If there is only one other bidder than what should Alice bid to maximize her profit? Show your work.
 - (b) (EXTRA CREDIT) If there are two other bidders than what should Alice bid to maximize her profit? Show your work.
 - (c) (EXTRA CREDIT) If there are n other bidders than what should Alice bid to maximize her profit? Show your work.

SOLUTION TO PROBLEM 2

a) Assume Alice bids x . Only one other bidder.

Prob of Alice WINNING is $\frac{x}{b}$. Prob of Alice TIEING is $\frac{1}{b}$. Hence Alice's expected profit is

$$\frac{x}{b}(v-x) + \frac{1}{b}\left(\frac{v}{2} - \frac{x}{2}\right) = \frac{1}{b}\left(x(v-x) + \frac{v-x}{2}\right) = \frac{1}{b}(v-x)\left(x + \frac{1}{2}\right).$$

We ignore the $\frac{1}{b}$ term and take the derivative using the product rule.

$$f'(x) = (v-x) - \left(x + \frac{1}{2}\right) = v - 2x - \frac{1}{2}$$

Set this to 0 to get $x = \frac{v+0.5}{2}$.

3. (50 points) Redo the PIRATES problem with the following change: The CAPTAIN gets two votes. Figure out the problem for $n = 1, 2, 3, \dots, 10$.

SOLUTION OMITTED