Examples of Three Person Cake Cutting With Uniform Valuations

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Credit Where Credit is Due

The paper

How to Cut a Cake Before the Party Ends

by

David Kurokawa, John K. Lai, Ariel Procaccia

has a protocol for envy-free cake cutting with piecewise linear valuations. Their paper inspired these slides.

We refer to their paper as ENDS.

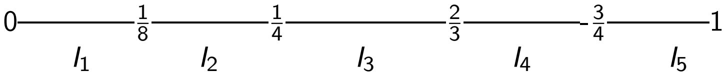
Alice, Bob, Carol

Alice's tastes are uniform on $[\frac{1}{8}, 1]$. Multiplier: $\frac{8}{7}$.

Bob's tastes are uniform on $[0, \frac{2}{3}]$. Multiplier: $\frac{3}{2}$.

Carol's tastes are uniform on $[\frac{1}{4}, \frac{3}{4}]$. Multiplier: 2.

Intervals



- How much of I_1 should Alice get?
- How much of I_1 should Bob get?
- How much of I_1 should Carol get?
- How much of I_2 should Alice get?
- How much of I_2 should Bob get?
- How much of I_2 should Carol get?
- Etc.

Variables

x_{1A} is how much Alice gets of I_1.
x_{1B} is how much Bob gets of I_1.
x_{1C} is how much Carol gets of I_1.
x_{2A} is how much Alice gets of I_2.
x_{2B} is how much Bob gets of I_2.
x_{2C} is how much Carol gets of I_2.
$\vdots$
x_{iP} is how much Person P gets of I_i.

NOTE: $x_{1A} = x_{4B} = x_{5B} = x_{1C} = x_{2C} = x_{5C} = 0$.

Example: $x_{2A} = \frac{1}{10} \rightarrow$ Alice gets subinterval of I_2 of length $\frac{1}{10}$.

Equations: The x_{iP} Make Sense

I_1 of length $\frac{1}{8}$: $\quad 0 \le x_{1B} \le \frac{1}{8} - 0 = \frac{1}{8}$

I_2 of length $\frac{1}{8}$: $\quad 0 \le x_{2A}, x_{2B} \le \frac{1}{4} - \frac{1}{8} = \frac{1}{8}$.

I_3 of length $\frac{5}{12}$: $\quad 0 \le x_{3A}, x_{3B}, x_{3C} \le \frac{2}{3} - \frac{1}{4} = \frac{5}{12}$

I_4 of length $\frac{1}{12}$: $\quad 0 \le x_{4A}, x_{4C} \le \frac{3}{4} - \frac{2}{3} = \frac{1}{12}$

I_5 of length $\frac{1}{4}$: $\quad 0 \le x_{5A} \le 1 - \frac{3}{4} = \frac{1}{4}$

We will not mention these again for a while.

Equations: The x_{iP} Make Sense

I_1 of length $\frac{1}{8}$: $x_{1B} = \frac{1}{8}$

I_2 of length $\frac{1}{8}$: $x_{2A} + x_{2B} = \frac{1}{8}$

I_3 of length $\frac{5}{12}$: $x_{3A} + x_{3B} + x_{3C} = \frac{5}{12}$

I_4 of length $\frac{1}{12}$: $x_{4A} + x_{4C} = \frac{1}{12}$

I_5 of length $\frac{1}{4}$: $x_{5A} = \frac{1}{4}$

We set

$$x_{1B} = \frac{1}{8} \qquad x_{5A} = \frac{1}{4}.$$

The first and fifth equation are now satisfied.

Equations: Getting Everyone $\geq \frac{1}{3}$

Alice gets $\geq \frac{1}{3}$: $\frac{8}{7}(x_{2A} + x_{3A} + x_{4A} + \frac{1}{4}) \geq \frac{1}{3}$

$$\frac{8}{7}(x_{2A} + x_{3A} + x_{4A}) \geq \frac{1}{21}$$

Bob gets $\geq \frac{1}{3}$: $\frac{3}{2}(\frac{1}{8} + x_{2B} + x_{3B}) \geq \frac{1}{3}$

$$\frac{3}{2}(x_{2B} + x_{3B}) \geq \frac{7}{48}$$

Carol gets $\geq \frac{1}{3}$:

$$2(x_{3C} + x_{4C}) \geq \frac{1}{3}$$

ALL the Equations

All vars ≥ 0.

$$\begin{aligned}
x_{2A} + x_{2B} &= \frac{1}{8} \\
x_{3A} + x_{3B} + x_{3C} &= \frac{5}{12} \\
x_{4A} + x_{4C} &= \frac{1}{12} \\
\frac{8}{7}(x_{2A} + x_{3A} + x_{4A}) &\geq \frac{1}{21} \\
\frac{3}{2}(x_{2B} + x_{3B}) &\geq \frac{7}{48} \\
2(x_{3C} + x_{4C}) &\geq \frac{1}{3}
\end{aligned}$$

Can solve by REASONING or by an LP package.

Reasoning:

- Give Carol first– she has largest multiplier.
- Give Carol from I_4, only Alice competes there.
- Giver her ALL of I_4 since still does not get Carol $\frac{1}{3}$.
- Recall:

$$\begin{aligned} x_{4A} + x_{4C} &= \frac{1}{12} \\ 2(x_{3C} + x_{4C}) &\geq \frac{1}{3} \end{aligned}$$

- Set $x_{4C} = \frac{1}{12}$. Forces $x_{4A} = 0$.
- $2(x_{3C} + \frac{1}{12}) \geq \frac{1}{3}$
- Set $x_{3C} = \frac{1}{6} - \frac{1}{12} = \frac{1}{12}$.
- Carol has $1/3$, Interval I_4 is allocated.

Making Bob Happy

Plugging in $x_{4A} = 0$, $x_{4C} = \frac{1}{12}$, $x_{3C} = \frac{1}{12}$ yields:

$$\begin{aligned} x_{2A} + x_{2B} &= \frac{1}{8} \\ x_{3A} + x_{3B} &= \frac{1}{3} \\ \frac{8}{7}(x_{2A} + x_{3A}) &\geq \frac{1}{21} \\ \frac{3}{2}(x_{2B} + x_{3B}) &\geq \frac{7}{48} \end{aligned}$$

Satisfy Bob: Give Bob from smaller interval I_2 (makes math easier) give him ALL of it: $x_{2B} = \frac{1}{8}$. Forces $x_{2A} = 0$.

Making Bob Happy

Plug in $x_{2B} = \frac{1}{8}$ and $x_{2A} = 0$.

$$\begin{aligned} x_{3A} + x_{3B} &= \tfrac{1}{3} \\ \tfrac{8}{7}(x_{3A}) &\ge \tfrac{1}{21} \\ \tfrac{3}{2}(\tfrac{1}{8} + x_{3B}) &\ge \tfrac{7}{48} \end{aligned}$$

Give Bob enough of I_2 so that he is happy:

$$\frac{1}{8} + x_{3B} \ge \frac{7}{72}$$

$$x_{3B} \ge \frac{55}{576}$$

Set $x_{3B} = \frac{55}{576}$. Forces $x_{3A} = \frac{1}{3} - \frac{55}{576} = \frac{137}{576}$. Does this work?

Final Reckoning

Alice: $x_{1A} = 0$, $x_{2A} = 0$, $x_{3A} = \frac{137}{576}$, $x_{4A} = 0$, $x_{5A} = \frac{1}{4}$.

$$\frac{8}{7}(0 + 0 + \frac{137}{576} + 0 + \frac{1}{4}) \sim 0.5575$$

Bob: $x_{1B} = \frac{1}{8}$, $x_{2B} = \frac{1}{8}$, $x_{3B} = \frac{55}{576}$, $x_{4B} = 0$, $x_{5B} = 0$.

$$\frac{3}{2}(\frac{1}{8} + 0 + \frac{1}{8} + \frac{55}{576} + 0 + 0) \sim 0.5182$$

Carol: $x_{1C} = 0$, $x_{2C} = 0$, $x_{3C} = \frac{1}{12}$, $x_{4C} = \frac{1}{12}$, $x_{5C} = 0$.

$$2(0 + 0 + \frac{1}{12} + \frac{1}{12} + 0) = \frac{1}{3} \sim 0.3333$$

TOTAL:

$$0.5575 + 0.5182 + 0.3333 = 1.409$$

MOST UNHAPPY: Carol with 0.33333.

The Linear Programming Problem Maximize (or Minimize) a LINEAR function relative to LINEAR constraints.

Example

Maximize

$$4x + 8y - 7z$$

Relative to

$$\begin{aligned} -3x + 5y - 8z &\le 20 \\ x + y + z &\le 5 \\ 2x + y + 18z &\le 100 \\ 7x + 29y + 178z &\le 193 \end{aligned}$$

- VERY practical problem. Many REAL applications.
- There are MANY PACKAGE for it that are easy to use: `http://www3.nd.edu/~jeff/mathprog/mathprog.html`

Linear Programming

We want $x_{2A}, x_{2B}, x_{3A}, x_{3B}, x_{3C}, x_{4A}, x_{4C}$ that satisfies:
$0 \le x_{2A}, x_{2B} \le \frac{1}{8}$
$0 \le x_{3A}, x_{3B}, x_{3C} \le \frac{5}{12}$
$0 \le x_{4A}, x_{4C} \le \frac{1}{12}$

$x_{2A} + x_{2B} = \frac{1}{8}$
$x_{3A} + x_{3B} + x_{3C} = \frac{5}{12}$
$x_{4A} + x_{4C} = \frac{1}{12}$

$\frac{8}{7}(x_{2A} + x_{3A} + x_{4A} + \frac{1}{4}) \ge \frac{1}{3}$
$\frac{3}{2}(\frac{1}{8} + x_{2B} + x_{3B}) \ge \frac{1}{3}$
$2(x_{3C} + x_{4C}) \ge \frac{1}{3}$

What to Maximize?- TOTAL Happiness

Our Goal is WEAKER than Linear Programming- all we want to do is find SOME point.
But can use this framework:
MAXIMIZE total happiness
or
MINIMIZE individual unhappiness

$$\frac{8}{7}(x_{2A} + x_{3A} + x_{4A} + \frac{1}{4}) + \frac{3}{2}(\frac{1}{8} + x_{2B} + x_{3B}) + 2(x_{3C} + x_{4C})$$

Maximizing Total Happiness

Plugged into an LP package:

A: $x_{1A} = 0$, $x_{2A} = 0.0277$, $x_{3A} = 0.0138$, $x_{4A} = 0$. $x_{5A} = 0.25$

$$\frac{8}{7}(0 + 0.0277 + 0.0138 + 0 + 0.25) = 0.333$$

B: $x_{1B} = 0.125$, $x_{2B} = 0.0972$, $x_{3B} = 0$, $x_{4B} = 0$, $x_{5B} = 0$.

$$\frac{3}{2}(0.125 + 0.0972 + 0 + 0 = 0) = 0.333$$

C: $x_{1C} = 0$, $x_{2C} = 0$, $x_{3C} = 0.403$, $x_{4C} = 0.083$, $x_{5C} = 0$.

$$2(0 + 0 + 0.403 + 0.083 + 0) = 0.972$$

TOTAL:

$$0.3333 + 0.3333 + 0.97222 = 1.638$$

MOST UNHAPPY: Alice and Bob 0.3333.

Minimize Unhappiness

Add a variable t.

$\frac{8}{7}(x_{2A} + x_{3A} + x_{4A} + \frac{1}{4}) \ge t$

$\frac{3}{2}(\frac{1}{8} + x_{2B} + x_{3B}) \ge t$

$2(x_{3C} + x_{4C}) \ge t$

Maximize t.

Minimizing Ind. Unhappiness

Plugged into an LP package:

A: $x_{1A} = 0$, $x_{2A} = 0$, $x_{3A} = 0.17857$, $x_{4A} = 0$. $x_{5A} = 0.25$

$$\frac{8}{7}(0 + 0 + .178587 + 0.25) = 0.4898$$

B: $x_{1B} = 0.125$, $x_{2B} = 0.125$, $x_{3B} = 0.076531$, $x_{4B} = 0$, $x_{5B} = 0$.

$$\frac{3}{2}(0.125 + 0.125 + 0.076531 + 0 + 0) = 0.4898$$

C: $x_{1C} = 0$, $x_{2C} = 0$, $x_{3C} = 0.16156$, $x_{4C} = 0.083$, $x_{5C} = 0$.

$$2(0 + 0 + 0.16156 + 0.083 + 0) = 0.4898.$$

TOTAL:

$$0.4898 + 0.4898 + 0.4898 = 1.4694$$

MOST UNHAPPY: ALL have 0.4898.

Protocol

Protocol for n players, all have uniform valuations.

1. Every player simul reveals their valuation. (honestly)
2. Players form LP program to satisfy that all have $\geq 1/n$, vars make sense, and total is maximized (OR to minimize Unhappiness). They solve the LP.
3. Player make the cuts as the LP solution dictates.

- How many cuts? $\leq 2n-1$ intervals, $\leq n-1$ cuts. PLUS the cuts at each interval, $\leq 2n-2$ cuts. TOTAL NUMBER OF CUTS: $\leq (2n-1)(n-1)+2n-2 = 2n^2-n-2$.
- Does this LP always have a solution? Yes.
- The paper ENDS has an $O(n^2)$ protocol for envy-free (hence prop) but does not maximize total. Extends to piece-wise valuations but with diff bound depending on number-of-pieces.

Can we make Division Envy-Free?

Inequalities for Envy Free:

Alice not envious of Bob: $x_{2A} + x_{3A} + x_{4A} + \frac{1}{4} \geq x_{2B} + x_{3B}$.

Alice not envious of Carol: $x_{2A} + x_{3A} + x_{4A} + \frac{1}{4} \geq x_{3C} + x_{4C}$.

Bob not envious of Alice: $\frac{1}{8} + x_{2B} + x_{3B} \geq x_{2A} + x_{3A}$

Bob not envious of Carol: $\frac{1}{8} + x_{2B} + x_{3B} \geq x_{3C}$

Carol not envious of Alice: $x_{3C} + x_{4C} \geq x_{3A} + x_{4A}$

Carol not envious of Bob: $x_{3C} + x_{4C} \geq x_{3B}$

All Constraints for Envy Free

$$
\begin{aligned}
x_{2A} + x_{2B} &= \frac{1}{8} \\
x_{3A} + x_{3B} + x_{3C} &= \frac{5}{12} \\
x_{4A} + x_{4C} &= \frac{1}{12} \\
x_{2A} + x_{3A} + x_{4A} + \frac{1}{4} &\ge x_{2B} + x_{3B} \\
x_{2A} + x_{3A} + x_{4A} + \frac{1}{4} &\ge x_{3C} + x_{4C} \\
\frac{1}{8} + x_{2B} + x_{3B} &\ge x_{2A} + x_{3A} \\
\frac{1}{8} + x_{2B} + x_{3B} &\ge x_{3C} \\
x_{3C} + x_{4C} &\ge x_{3A} + x_{4A} \\
x_{3C} + x_{4C} &\ge x_{3B}
\end{aligned}
$$

Final Reckoning- Envy Free

Maximize Total:

Alice: $x_{1A} = 0$, $x_{2A} = 0$, $x_{3A} = 0.1111$, $x_{4A} = 0$, $x_{5A} = 0.25$.

$$\frac{8}{7}(0 + 0 + 0.1111 + +0 + 0 + 0.25) \sim 0.4126$$

Bob: $x_{1B} = 0.125$, $x_{2B} = 0.125$, $x_{3B} = 0.02777$, $x_{4B} = 0$, $x_{5B} = 0$.

$$\frac{3}{2}(0.125 + 0.125 + 0.02778 + 0 + 0) \sim 0.41667$$

Carol: $x_{1C} = 0$, $x_{2C} = 0$, $x_{3C} = 0.2777$, $x_{4C} = 0.08333$, $x_{5C} = 0$.

$$2(0 + 0 + 0.2777 + 0.08333) \sim 0.722$$

TOTAL:

$$0.4162 + 0.4166 + 0.722 = 1.5512$$

MOST UNHAPPY: Alice with 0.4126.

Minimize Unhappiness

Got same numbers as wanted just proportional and min unhappiness.

Protocol

Envy Free Protocol for n players, all have uniform valuations.

1. Every player simul reveals their valuation. (honestly)
2. Players form LP program to satisfy that there is no envy, all vars make sense, and total is maximized. (They set the obv vars to 0 and whatever else is forced.) They solve the LP.
3. Player make the cuts as the LP solution dictates.

- How many cuts? As before $\le 2n^2 - n - 2$.
- Does this LP always have a solution? Yes.
- The paper ENDS has an $O(n^2)$ protocol for envy-free (hence prop) but does not maximize total. Extends to piece-wise valuations but with diff bound depending on number-of-pieces.

Other Valuations

What if Valuation is of
$v(c,d) = \int_c^d (ax+b)dx = \frac{a}{2}(d^2 - c^2) + b(d-c)$.

Only makes sense if $1 = v(0,1) = \int_0^1 (ax+b)dx = \frac{a}{2} + b$.

$1 = \frac{a}{2} + b$

We do an example.

Example

Let $f(x) = 2x$, $g(x) = x + \frac{1}{2}$, $h(x) = \frac{x}{2} + \frac{3}{4}$.

Alice's Val: $val_A(b,a) = \int_a^b f(x) = b^2 - a^2$.

Bob's Val: $val_B(b,a) = \int_a^b g(x) = \frac{1}{2}(b^2 - a^2) + \frac{1}{2}(b-a)$.

Carol's Val: $val_C(b,a) = \int_a^b h(x) = \frac{1}{4}(b^2 - a^2) + \frac{3}{4}(b-a)$.

Note: $f(x), g(x), h(x)$ all MEET at $(\frac{1}{2}, 1)$.

Intervals

This is DIFF than before.

$$\begin{array}{ccccccccccccc} 0 & -- & x_1 & -- & x_2 & -- & \frac{1}{2} & -- & x_3 & -- & x_4 & -- & 1 \\ & C & & B & & A & & A & & B & & C & \end{array}$$

- A gets $[x_2, \frac{1}{2}] \cup [\frac{1}{2}, x_3]$
- B gets $[x_1, x_2] \cup [x_3, x_4]$
- C gets $[0, x_1] \cup [x_4, 1]$

Who Gets What?

$$0 -- \quad x_1 \quad -- \quad x_2 \quad -- \quad \frac{1}{2} \quad -- \quad x_3 \quad -- \quad x_4 \quad -- \quad 1$$
$$C \qquad B \qquad A \qquad A \qquad B \qquad C$$

A gets

$$\left(\frac{1}{2}\right)^2 - x_2^2 + x_3^2 - \left(\frac{1}{2}\right)^2 = x_3^2 - x_2^2$$

B gets

$$\frac{1}{2}(x_2^2 - x_1^2 + x_4^2 - x_3^2) + \frac{1}{2}(x_2 - x_1 + x_4 - x_3)$$

C gets

$$\frac{1}{4}(x_1^2 + 1 - x_4^2) + \frac{3}{4}(x_1 + 1 - x_4)$$

Alice's View of the World

Alice thinks:

Alice gets $x_3^2 - x_2^2$

Bob gets $x_2^2 - x_1^2 + x_4^2 - x_3^2$

Carol gets $x_1^2 + 1 - x_4^2$.

Equations so that Alice has no envy:

$x_3^2 - x_2^2 \geq x_2^2 - x_1^2 + x_4^2 - x_3^2$

$x_3^2 - x_2^2 \geq x_1^2 + 1 - x_4^2$.

Bob's View of the World

Bob thinks:

Alice gets $\frac{1}{2}(x_3^2 - x_2^2) + \frac{1}{2}(x_3 - x_2)$

Bob gets $\frac{1}{2}(x_2^2 - x_1^2 + x_4^2 - x_3^2) + \frac{1}{2}(x_2 - x_1 + x_4 - x_3)$

Carl gets $\frac{1}{2}(x_1^2 + 1 - x_4^2) + \frac{1}{2}(x_1 + 1 - x_4)$

Equations so that Bob has no envy:

$(x_2^2 - x_1^2 + x_4^2 - x_3^2) + (x_2 - x_1 + x_4 - x_3) \geq (x_3^2 - x_2^2) + (x_3 - x_2)$

$(x_2^2 - x_1^2 + x_4^2 - x_3^2) + (x_2 - x_1 + x_4 - x_3) \geq (x_1^2 + 1 - x_4^2) + (x_1 + 1 - x_4)$

Carol's View of the World

Carol thinks:

Alice gets $\frac{3}{4}(x_3^2 - x_2^2) + \frac{1}{4}(x_3 - x_2)$

Bob gets $\frac{3}{4}(x_2^2 - x_1^2 + x_4^2 - x_3^2) + \frac{1}{4}(x_2 - x_1 + x_4 - x_3)$

Carol gets $\frac{3}{4}(x_1^2 + 1 - x_4^2) + \frac{1}{4}(x_1 + 1 - x_4)$

Equations so that Bob has no envy:

$$3(x_1^2 + 1 - x_4^2) + (x_1 + 1 - x_4) \geq 3(x_3^2 - x_2^2) + (x_3 - x_2)$$

$$3(x_1^2+1-x_4^2)+(x_1+1-x_4) \geq 3(x_2^2-x_1^2+x_4^2-x_3^2)+(x_2-x_1+x_4-x_3)$$

Problem 1:

Problem 1: Does there exist x_1, x_2, x_3, x_4 that satisfies the following inequalities:

$0 \le x_1 \le x_2 \le x_3 \le x_4 \le 1$

$x_3^2 - x_2^2 \ge x_2^2 - x_1^2 + x_4^2 - x_3^2$
$x_3^2 - x_2^2 \ge x_1^2 + 1 - x_4^2$.

$(x_2^2 - x_1^2 + x_4^2 - x_3^2) + (x_2 - x_1 + x_4 - x_3) \ge (x_3^2 - x_2^2) + (x_3 - x_2)$
$(x_2^2 - x_1^2 + x_4^2 - x_3^2) + (x_2 - x_1 + x_4 - x_3) \ge (x_1^2 + 1 - x_4^2) + (x_1 + 1 - x_4)$

$3(x_1^2 + 1 - x_4^2) + (x_1 + 1 - x_4) \ge 3(x_3^2 - x_2^2) + (x_3 - x_2)$
$3(x_1^2 + 1 - x_4^2) + (x_1 + 1 - x_4) \ge 3(x_2^2 - x_1^2 + x_4^2 - x_3^2) + (x_2 - x_1 + x_4 - x_3)$

Note: Can Phrase as Quad Prog Problem.

Quadratic Programming

The Quadratic Programming Problem Maximize (or Minimize) a LINEAR function relative to QUADRATIC constraints.

Example

Maximize

$$4x + 8y - 7z$$

Relative to

$$\begin{aligned}
-3x^2 + 5y - 8z^2 &\le 20 \\
x^2 + y^2 + z &\le 5 \\
2x + y^2 + 18z &\le 100 \\
7x + 29y + 178z^2 &\le 193
\end{aligned}$$

- NP-Hard. Thought to be HARD.
- There is ONE PACKAGES for it that I know.

Problem 2: Maximize

$$\left(\frac{1}{2}\right)^2 - x_2^2 + x_3^2 - \left(\frac{1}{2}\right)^2 + x_3^2 - x_2^2 + \frac{1}{2}(x_2^2 - x_1^2 + x_4^2 - x_3^2) + \frac{1}{2}(x_2 - x_1 + x_4 - x_3)$$

$$+\frac{1}{4}(x_1^2 + 1 - x_4^2) + \frac{3}{4}(x_1 + 1 - x_4)$$

while satisfying:

$0 \le x_1 \le x_2 \le x_3 \le x_4 \le 1$

$x_3^2 - x_2^2 \ge x_2^2 - x_1^2 + x_4^2 - x_3^2$

$x_3^2 - x_2^2 \ge x_1^2 + 1 - x_4^2.$

$(x_2^2 - x_1^2 + x_4^2 - x_3^2) + (x_2 - x_1 + x_4 - x_3) \ge (x_3^2 - x_2^2) + (x_3 - x_2)$

$(x_2^2 - x_1^2 + x_4^2 - x_3^2) + (x_2 - x_1 + x_4 - x_3) \ge (x_1^2 + 1 - x_4^2) + (x_1 + 1 - x_4)$

$3(x_1^2 + 1 - x_4^2) + (x_1 + 1 - x_4) \ge 3(x_3^2 - x_2^2) + (x_3 - x_2)$

$3(x_1^2 + 1 - x_4^2) + (x_1 + 1 - x_4) \ge 3(x_2^2 - x_1^2 + x_4^2 - x_3^2) + (x_2 - x_1 + x_4 - x_3)$

NOT a Quad Programming Problem

We want to maximize a **Quadratic function** relative to **Quadratic Constraints**. We call this **Quadratic Quadratic Programming** (QQP).

QQP has not been studied. **Rumors** of a packages that **might** solve it.

SOOL? FML? FUBAR?

FML!!! My prof wants me to solve a QQP!!!

Protocol

Envy Free Protocol for n players, all have linear valuations.

1. Every player simul reveals their valuation. (honestly)
2. Players form QQP program to satisfy that there is no envy, all vars make sense, and total is maximized. Solve the QQP.
3. If someone starves to death while solving the QQP then remove them and re-do equations. Repeat if needed.
4. If there are ≥ 2 people left when solved then use the solution. If there is only 1 person left, he gets it.

Serious Protocol and Open Questions

Envy Free Protocol for n players, all have linear valuations.

1. Every player simul reveals their valuation. (honestly)
2. Players form QQP program to satisfy that there is no envy, all vars make sense, and total is maximized. Solve the QQP.
3. Solve it.
4. Cut the cake as it dictates.

- Does a QQP of his form always have a solution?
- Is there always a rational point that satisfies the constraints? Unlikely.
- Is there an efficient algorithm to find an approx solution to the QQP that arise from this problem? (Do not know?)
- Will these be solved before or after the Gov. Shutdown ends?