

# Definitions of Fairness and Properties of Protocols

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# Credit Where Credit is Due

**NONE of this is my work**

# Ground Rules

People  $A_1, \dots, A_n$  are going to split an item.

- ▶ It could be a cake.
- ▶ It could be a set of items (e.g., use AW protocol).

# Valuations

Each player  $A_i$  has a valuation  $V_i$  of how they value parts of the item.

- ▶ The entire item is worth 1.
- ▶ Item is  $[0, 1]$ . There could be a function  $f$  such that

$$V_i(a, b) = \int_b^a f(x) dx.$$

Note that need  $\int_0^1 f(x) dx = 1$ .

- ▶ If item is a set of items could be  $V_i(PICASSO) = 0.75$  and  $V(CAR) = 0.25$ .

# What is Fair?

A division is  $(P_1, \dots, P_n)$  where  $A_i$  gets  $P_i$ .

1. A division is *Proportional* if  $(\forall i)[V_i(P_i) \geq \frac{1}{n}]$ .  
All think they got  $\geq \frac{1}{n}$ .
2. A division is *Envy-Free* if  $(\forall i, j)[V_i(P_i) \geq V_i(P_j)]$ .  
All think they got the biggest piece (or tied).
3. A division is *Equitable* if  $(\forall i, j)[V_i(P_i) = V_j(P_j)]$ .  
All got exact same size piece.

# What is Efficient?

## Definition

An division  $(P_1, \dots, P_n)$  is *better than* a division  $(P'_1, \dots, P'_n)$  if some player does better and no player does worse.

## Definition

A division is *Efficient* if there is no better division.

# What is a Protocol

A *protocol* will have

1. Instructions that a player can carry out that the others can easily verify.
2. Advice for the player which he need not follow.

## Definition

If a Player does not follow the advice then he *cheats*.

# Cheaters, Quitters, Winners

We want to prove that

1. Cheaters never win. (Except the New England Patriots.)
2. Winners never cheat. (Except the New England Patriots.)
3. Quitters never win. (By Definition)
4. Winners never quit. (After the win— why not?)
5. Cheaters never quit. (what?)
6. Quitters never cheat. (By Definition)

# Example of a Protocol

## Cut and Choose

1. Alice cuts a pie in half (equal in her eyes)
2. Bob picks one of those pieces (the bigger one in his eyes)

Is easy to show that if Alice cheats there is a scenario where she does worse than if she was honest.

# What is a Good Protocol?

## Definition

1. A protocol is *cheat proof* if for all ways that a player can cheat, there is a scenario where he will get LESS than if he played honestly.
2. A protocol is *super cheat proof* if even if a player knows the others players preferences, cheating may lead to him getting less than if he was honest.
3. A protocol is *proportional cheat proof* if even if a player cheats, the rest get  $\geq \frac{1}{n}$ .
4. A protocol is *envy-free cheat proof* if even if a player cheats, the rest all think they got the biggest piece (or tied).