1. (30 points) Consider five candidates (a,b,c,d,e) and seven voters with the following preferences:

| c | d | с | b | е | d | с |
|---|---|---|---|---|---|---|
| a | a | e | d | d | е | a |
| e | e | d | a | a | a | е |
| b | c | a | e | c | b | b |
| d | b | b | с | b | с | d |

For each of the six voting systems,

- (a) Condorcet
- (b) Plurality
- (c) Borda count
- (d) Hare system (Single transferable vote)
- (e) Sequential pairwise voting (with ordering a,b,c,d,e)
- (f) Dictatorship (with fourth voter as Dictator)

determine the winner of the election (or state that there is none). If there are ties, indicate all of the winners. For Borda count give the count for each candidate. For Hare system indicate the order that the candidates are eliminated. For Sequential pairwise voting indicate the order of the pairwise contests and the winners.

- 2. (30 points) Consider a voting system where first you check if there is a Condorcet winner and if so that candidate is the winner. If not, you select the winner using the Borda count. For each of the following state whether it satisfies the condition. If so justify; if not give a counterexample.
  - (a) Condorcet winner
  - (b) Pareto
  - (c) Monotonicity
  - (d) Independence of irrelevant alternatives
- 3. (40 points) Show that a voter can manipulate the outcome of an election using the Hare System (Single Transferable Vote) by submitting an insincere ballot.