1. (0 points) What is your name? Write it clearly. Staple your HW. When is the midterm? When is the final?

2. (30 points) Assume that in his PhD Thesis Karthik came up with a protocol that for unfair division that does the following:

   If $a \leq 200$ or $b \leq 300$ then the following hold:

   - If a, b are both odd then can do unfair division for $(a : b)$ in $\left\lceil \sqrt{\lceil \log_2(a+b) \rceil} \right\rceil$ cuts.

   - If $a < b$, a is even, and b is odd, then can do unfair division for $(a : b)$ in $\left\lceil \log_2 \left\lceil \sqrt{(a+b)} \right\rceil \right\rceil$ cuts.

   - If $a < b$, a is odd, and b is even, then can do unfair division for $(a : b)$ in $\left\lceil \log_2 \left\lceil \sqrt{(a+b)} \right\rceil \right\rceil$ cuts.

   Alice and Bob want to do unfair division on $(200 : 301)$. The ONLY cuts Alice will consider are

   $[25 : 476]$ (Either Alice or Bob gets the 25-piece)

   $[75 : 426]$ (Either Alice or Bob gets the 75-piece)

   $[125 : 376]$ (Either Alice or Bob gets the 125-piece)

   $[175 : 326]$ (Either Alice or Bob gets the 175-piece)

   $[225 : 276]$ (Either Bob gets the 225-piece or Bob gets the 276-piece)

   $[250 : 251]$ (Either Bob gets the 250-piece or Bob gets the 251-piece)

   Note that in all cases you only go back one steps since Karthik's algorithm will take over from there.

   Of the cuts Alice is willing to make, which will yield the min number of total cuts (it could be a tie).

3. (30 points) Alice and Bob want to split a cake in ratio $(3 : 5)$. They will use the near-halves algorithm.

   (a) Draw out the entire near-halves protocol as a tree. You may stop when a leaf is at $(1 : a)$ or $(a : 1)$. Write down next to each leaf how many cuts it took.

   (b) Give an example of when Alice knows Bob's tastes and uses that to get more than $3/8$ of the cake.

4. (40 points) Show that for all $(a : b)$, for all ϵ, there is a protocol such that *even if Alice and Bob know each others tastes* at the end Alice has within ϵ of $\frac{a}{a+b}$ of the cake, and Bob has within ϵ of $\frac{b}{a+b}$ of the cake.

5. (EXTRA CREDIT- FOR FUN AND TO GET A GOOD LETTER OUT OF ME) Find an infinite sequence of pairs (a_n, b_n) such that a_n and b_n have not common factors and there is a way to do unfair division $(a_n : b_n)$ with $\leq \frac{1}{3} \lceil \log_2(a_n + b_n) \rceil$. Then try to improve the $\frac{1}{3}$ to even lower constants. The examples must be simple enough to present to the class.