HON 209M, Midterm

Do not open this exam until you are told. Read these instructions:

1. This is a closed book exam, though ONE sheet of notes is allowed. **You may use a Calculators, or other aids are allowed**. If you have a question during the exam, please raise your hand.
2. **You must turn in your exam immediately when time is called at the end.**
3. There are 4 problems which add up to 100 points. The exam is 1 hour and 30 minutes.
4. In order to be eligible for as much partial credit as possible, show all of your work for each problem, **write legibly**, and **clearly indicate** your answers. Credit **cannot** be given for illegible answers.
5. After the last page there is paper for scratch work. If you need extra scratch paper **after** you have filled these areas up, please raise your hand. Scratch paper must be turned in with your exam, with your name and ID number written on it, but scratch paper **will not** be graded.
6. Please write out the following statement: "*I pledge on my honor that I will not give or receive any unauthorized assistance on this examination.*"

7. Fill in the following:

NAME :
SIGNATURE :
SID :
SECTION NUMBER :

SCORES ON PROBLEMS

Prob 1:	
Prob 2:	
Prob 3:	
Prob 4:	
TOTAL	

1. (30 points) For each of the following mark the statement as TRUE, FALSE, or UNKNOWN TO SCIENCE. 3 points for a correct response, -2 for an incorrect response. (ADVICE: Do not guess!)

   (a) For all $n \geq 2$ there is a discrete n-person protocol for proportional fair division that takes $\leq 100n \log_2 n$ cuts.

   (b) For all $n \geq 2$ there is a discrete n-person protocol for proportional fair division that takes $\leq 100n$ cuts.

   (c) For all $n \geq 2$ there is a discrete n-person protocol for envy-free fair division that takes $\leq 100n^3$ cuts.

   (d) For all $\epsilon > 0$, for all $n \geq 3$ there is a discrete n-person protocol such that, at the end, (1) everyone has within ϵ of $1/n$, and (2) one person has exactly $1/n$.

   (e) For all $n \geq 3$ there is a discrete n-person protocol such that, at the end, everyone has exactly $1/n$.

   (f) There exists a, b such that any discrete 2-person protocol for $(a : b)$ division requires at least 10^{100} steps.

   (g) There exists a, b such that any discrete 2-person protocol for $(a : b)$ division requires at least ab steps.

   (h) There is a discrete 3-person protocol for proportional cake cutting that uses at most 3 cuts.

   (i) There is a discrete 4-person protocol for proportional cake cutting that uses at most 4 cuts.

   (j) There is a discrete 5-person protocol for proportional cake cutting that uses at most 5 cuts.

2. (20 points) Consider the following protocol for dividing up a finite set of goods, some of which are fluid (can be split).

   (a) Alice and Bob agree on an ordering of the items which is NOT in terms of its value but in terms of which ones they are happiest splitting. For example, if the items were MONEY, GOLD, CAR then they MOST prefer to split MONEY, then second favorite to split is GOLD, but last favorite to split is CAR. So they would output (MONEY,GOLD,CAR).

   (b) Alice and Bob both allocate point values for all of the goods that add up to 100 (identical to the first stage of the Adjusted Winner Protocol). They CANNOT give two goods the same value.

   (c) The good are given out using ABBAABBAABBA.... That is, Alice gets the good that she values most, then Bob gets the good he values most that is available, then Bob gets the good that he values most that is available, then Alice gets the good that she values most that is available.

   (d) Alice and Bob both determine how many points worth of goods they got. We call the one who is better off *the better-off one*

   (e) If they do not have the same number of points then the good they are happiest splitting that the better-off one has is the one they split (similar to the Adjusted Winner Protocol).

   (The next two pages have two problems that use this protocol.)

Problem 1.1) Use this protocol in the following scenario: All items are splittable. Their preference for splitting is (Money,Gold,Ice Cream, Jewelery) meaning that they are happiest splitting Money and least happiest splitting jewelery.

Specify what each person gets and how many points they get.

Item	Alice	Bob
Money	40	40
Gold	30	20
Jewelery	20	10
Ice Cream	10	30

SOLUTION TO PROBLEM 1.1

Alice goes first and gets MONEY.

Bob goes second and gets ICE CREAM.

Bob goes third and gets GOLD

Alice goes last and gets JEWELERY

ALICE TOTAL: 40+20=60

BOB TOTAL: 30+20=50

So Alice needs to give Bob some good. They will use Money since they are happy splitting it and Alice has that one.

Need x such that

$60 - 40x = 50 + 40x$

$10 = 80x$

$x = 1/8$.

So Alice gives Bob 1/8 of her money.

They both get 50 + 40/8 = 50 + 5 = 55.

Problem 1.2) Assume Bob KNOWS Alice's point allocation. Say how he should dishonestly present his points so that he gets MORE than he got in part (1).

SOLUTION TO 1.2

Item	Alice	Bob
Money	40	40
Gold	30	20
Jewelery	20	15
Ice Cream	10	25

Alice gets MONEY

Bob gets ICE CREAM

Bob gets GOLD

Alice gets JEWELERY

Alice gets 60

Bob gets 35

Alice gives Bob some money:

$60 - 40x = 35 + 40x$

$25 = 80x$

$x = 25/80 = 5/16$

So it LOOKS LIKE They both get 60 - 40*5/16 = 60 - 25/2 = 60-12.5 = 47.5

BUT in reality Bob has 50 + 40*5/16 = 50 + 12.5 = 62.5.

3. (30 points) For each of the following six situations either give a scenario where it happens or give a short explanation of why it can never happen. (there will be three on this page and two on the next page.

   (a) In the 3-person COME LATE protocol with Alice and Bob initially splitting the cake, Alice and Bob are envious OF EACH OTHER.

   SOLUTION TO 2.1.

   Alice and Bob split the cake so they both have 1/2.

   Carol and Alice: Carol splits it into thirds. Alice thinks this is a evenly split. Bob thinks its split 1/2-1/2-0 (so in terms of the entire cake 1/4-1/4-0). Bob thinks Alice got 1/2, Carol got 0 from Alice (though we'll see later 1/6) Alice thinks she got 1/3.

   Carol and Bob: Carol splits it into thirds. Bob thinks this is a evenly split. Alice thinks its split 1/2-1/2-0 (so in terms of the entire cake 1/4-1/4-0). Alice thinks Bob got 1/2, Carol got 0 (though 1/6 total) Alice thinks she got 1/3.

   (b) In the 3-person COME LATE protocol with Alice and Bob initially splitting the cake, Carol is envious of BOTH Alice and Bob.

   SOLUTION TO 2.2

   This cannot happen. Initially Carol thinks Alice got A and Bob got B where $A+B=1$. Hence she MUST think that at least one of them got $\leq 1/2$. We can assume that she thinks Alice got $\leq 1/2$. After Alice and Carol go through their phase of the protocol, Carol will think Alice got $\leq 1/3$.

   (c) In the 4-person TRIM protocol there is a person who envies EVERYONE else.

   SOLUTION TO 2.3

   This cannot happen. If (say) Alice thinks that Bob, Carol, Donna ALL got $> 1/4$ then she will think she got $< 1/4$.

(d) In the 4-person Trim protocol there are two players who are envious of each other.

SOLUTION TO 2.4

This cannot happen. If Alice gets a piece BEFORE Bob does then Bob thinks (1) Alice got $\leq 1/4$, and (2) Whats left is $\geq 3/4$, and (3) Bob will get at least 1/3 of whats left, so at least 1/4.

(e) In the 4-person Divide and Conquer protocol there are two players who are envious of each other.

SOLUTION TO 2.5

This CAN happen. Lets say that after the first phase Alice and Bob are splitting LEFT and Carol and Donna are splitting RIGHT. Alice can think that Carol is NUTS and that Donna got most of RIGHT, say 1/3 total. At the same time, Donna can think that Bob is NUTS and that Alice got LOTS of Left, say $\geq 1/3$.

4. (20 points) Show that, for all $\epsilon > 0$ there is a SIX-person protocol that will, given a cake, obtain an envy-free division of ALL BUT ϵ of it.

   SOLUTION TO PROBLEM 4

   This is FIRST PLAYER HAPPY protocol:

   (a) A divides the cake into 17 equal pieces (evenly).
   (b) B creates a 9-way tie- trims at most 8 pieces.
   (c) C creates a 5-way tie- trims at most 4 pieces.
   (d) D creates a 3-way tie- trims at most 2 pieces.
   (e) E creates a 2-way tie- trims at most 1 piece.
   (f) F picks.
   (g) E picks (must take one he trimmed if available).
   (h) D picks (must take one she trimmed if available).
   (i) C picks (must take one she trimmed if available).
   (j) B picks (must take one he trimmed if available).
   (k) A picks

   We now use FIRST-PLAYER HAPPY over and over again using different people for FIRST PLAYER.

Scratch Paper