Truth Values and Bits

Standard Convention: TRUE is 1, FALSE is 0

Standard Convention: n-inputs viewed as n-bit number.

Prop Formula is now **Boolean Formula**

Redo TTs

Basic Gates

Definition

1. *AND GATES:* k Inputs and outputs AND of them.
2. *OR GATES:* k Inputs and outputs OR of them.
3. *XOR GATES:* k Inputs and outputs XOR of them.
4. *NOT GATES:* 1 Inputs and outputs NEG of it.
5. Such devices **really** exist!
6. Draw on board.

Why AND, OR, XOR?

Discuss why Gates for AND, OR, XOR?

What is a Circuit?

Definition

A **Circuit** has n Boolean Inputs which feed into gates and at the end have one or more outputs.

Do Examples

Building a Circuit for a 1-Bit Adder

Definition

A *1-Bit Adder* is a circuit that takes 2 bits and outputs their sum (which will be 2 bits).

Make TT for 1-bit Adder

New Problem

1. Old Problem: Given a Fml, find the TT for it.
2. New Problem: Given a TT, find a Fml for it.
3. New Problem: Given a Fml, find a Circuit for it. (Easy)

Given a TT...

1. For each row R of the TT that returns 1 (or T):
   For each variable x_i
   $L_i = x_i$ if Row R has $x_i = T$;
   $L_i = \neg x_i$ if Row R has $x_i = F$.
2. Write down mini-fml $L_1 \wedge \cdots \wedge L_n$
   KEY: This mini-fml is true IFF that row happens.
3. Output the OR of all of the minifmls.

Do Example

The 3-Bit Adder...

We do the following

1. Write a TT for a 3-bit adder. Input is a 3-bit number **in base 2**.
2. Write a formula for the 4-outputs of a 3-bit adder.
3. Draw a circuit for the 3-bit adder
4. Easy to generalize?

Half-Adders and Full-Adders. . .

Definition

1. A *Half Adder (HA)* is a circuit that has 2 boolean inputs, 2 boolean outputs, and outputs the sum and the carry.

   Get Circuit For a HA

2. A *Full Adder (FA)* is a circuit that has 3 boolean inputs, 2 boolean outputs, and outputs the sum and the carry.

   Get Circuit For a FA

Using HA's and FA's build an n-bit Adder