

Homework 2, MORALLY Due Feb 10  
FOR THIS ENTIRE HW THE VARIABLES ARE 0-1 VALUED.

1. (0 points but you have to answer) What is your name? Write it clearly. Staple your HW.
2. (10 points) When is the first midterm (give Date and Time)? When is the second midterm (give Date and Time)? When is the final (give Date and Time)? By when do you have to inform Professor Gasarch that you cannot make the timeslot of the first or second midterm?
3. (40 points) Consider the following arithmetic function:

$$f(x_1, x_2, x_3, x_4, x_5, x_6) = \begin{cases} 1 & \text{if exactly ONE of the inputs is 1} \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

- (a) How many rows are in the TT for  $f$ ?
  - (b) Do you want to do the TT for  $f$  (hint: The answer is NO!!!!)
  - (c) Describe a way to build the circuit for  $f$  WITHOUT doing the TT for  $f$ .
  - (d) Use your method to construct the circuit for  $f$ . (That is, DRAW IT.)
  - (e) You used a trick to avoid writing down that TT. Name a function  $g(x_1, \dots, x_n)$  where the trick would save you LOTS of time.
  - (f) Name a function  $h(x_1, \dots, x_n)$  where the trick would NOT save LOTS of time.
4. (30 points)
- (a) Assume (as we have been) that we use only AND, OR, and NOT gates. How many gates are needed for a HA? for a FA? How many gates do you need to build an  $n$ -bit adder ?
  - (b) Assume that we can use AND, OR, NOT, and XOR gates. How many gates are needed for a HA? for a FA? How many gates to you need to build an  $n$ -bit adder ?

5. (30 points) We have been using unbounded fan-in gates— the AND and OR and XOR have as many wires going into them as we want. We call an AND gate with exactly  $k$  inputs a  $k$ -AND gate.
- (a) I want to build a 3-input AND gate out of 2-input AND gates. How many 2-input AND gates will this take?
  - (b) I want to build a 4-input AND gate out of 2-input AND gates. How many 2-input AND gates will this take?
  - (c) For  $5 \leq n \leq 10$ : I want to build a  $k$ -AND gate out of 2-AND gates. How many 2-AND gates will this take?
  - (d) Make a conjecture of the following form: If you want to build an  $n$ -AND gate out of 2-AND gates. then it will take  $XXX(n)$  2-AND gates.
6. (0 points but thing about— it will be discussed in recitation). Assume  $k \ll n$ . Roughly how many  $k$ -AND gates are needed to construct an  $n$ -AND gate?