Homework 4, MORALLY Due Feb 23

NOTE: DUE MONDAY FEB 23 IN RECITATION. IF YOUR CAT DIES THEN WED FEB 25 IN RECITATION.

1. (0 points but you have to answer) What is your name? Write it clearly. Staple your HW.

2. (30 points) Find a set X such that the following is true (and prove it).

   - $X \subseteq \{0, 1, 2, 3, 4, 5, 6, 7\}$
   - For all $n \in \mathsf{N}$, there exists $a \in X$ such that $n^2 \equiv a \pmod{8}$.
   - For all $a \in X$, there exists $n \in \mathsf{N}$ such that $n^2 \equiv a \pmod{8}$.

3. (30 points) Show that if $n \equiv 7 \pmod{8}$ then n CANNOT be written as the sum of three squares. (HINT: use the last problem.)

4. (20 points) Compute the following the smart way. Show all work and do not use a calculator.

   (a) $3^{1000000000000000} \pmod{7}$

   (b) $7^{1000000000000000} \pmod{13}$

5. (20 points) You learned that for p *prime* $a^p \equiv a \pmod{p}$. In this problem we will try to find what happens for non-primes.

   (a) Find a number L such that for all $a \in \{0, 1, 2, 3\}$, $a^L \equiv a \pmod{4}$.

   (b) Find a number L such that for all $a \in \{0, 1, 2, 3, 4, 5\}$, $a^L \equiv a \pmod{6}$.

   (c) (Optional) Make a conjecture about, for n NOT prime, what is the L such that $a^L \equiv a \pmod{n}$.