

Homework 8 REALLY DUE WED April 15 IN RECITATION

READING: READ the paper on the course website about the recip problem, which is on the website next to where you found THIS hw. I guarantee you that the paper is NOT a hoax.

1. (0 points but you have to answer) What is your name? Write it clearly. Staple your HW. When is the final?
2. (20 points) How many ways are there to rearrange the letters in **MISSISSIPPI** into DIFFERENT words? Express your answer in terms of factorials.
3. (30 points) Let $p(x) = (x+y+z)^{100}$. What is the coefficient of $x^{60}y^{30}z^{10}$ in $p(x)$? Express your answer in terms of factorials.
4. (30 points) Find FOUR numbers $D \geq 1000$ such that the following is true, and prove it:

For all $n \geq 6$ there exists $d_1 < d_2 < \dots < d_n$ such that

- $1 = \frac{1}{d_1} + \frac{1}{d_2} + \dots + \frac{1}{d_n}$, AND
- $d_n \equiv 0 \pmod{D}$.

5. (20 points) For each of the following either say TRUE and prove it or say LOOKS HARD, which means that neither you nor Bill nor Ioana can prove it. You may use the paper on recip problems on the class website.

(a) *For all $n \geq 5$ there exists $d_1 < d_2 < \dots < d_n$ such that*

$$1 = \frac{1}{d_1} + \frac{1}{d_2} + \dots + \frac{1}{d_n}, \text{ AND } d_n \equiv 0 \pmod{43}.$$

(b) *For all $n \geq 5$ there exists $d_1 < d_2 < \dots < d_n$ such that*

$$1 = \frac{1}{d_1} + \frac{1}{d_2} + \dots + \frac{1}{d_n}, \text{ AND } d_n \equiv 0 \pmod{600}.$$

(c) *For all $n \geq 5$ there exists $d_1 < d_2 < \dots < d_n$ such that*

$$1 = \frac{1}{d_1} + \frac{1}{d_2} + \dots + \frac{1}{d_n}, \text{ AND } d_n \equiv 0 \pmod{5}.$$

(d) *For all $n \geq 5$ there exists $d_1 < d_2 < \dots < d_n$ such that*

$$1 = \frac{1}{d_1} + \frac{1}{d_2} + \dots + \frac{1}{d_n}, \text{ AND } d_n \equiv 0 \pmod{105}.$$