Homework 12 REALLY Due May 12
NOTE- MAY 12 is THE LAST DAY OF CLASS

1. (0 points) Where and when is the final?

2. (30 points) Give a clean statement of the form

   PI wins $NIM(1,k)$ *on* n *stones iff* XXX

   where XXX depends on k and n.

   (HINT: Work out the pattern for $NIM(1,2)$, $NIM(1,3)$, $NIM(1,4)$, $NIM(1,5)$, $NIM(1,6)$ until you see a pattern of patterns. Start at 0.)

   SOLUTION TO PROBLEM TWO

   For a NIM game let $W(n)$ be who wins if you begin with n on the board. Recall that

   $W(0) = II$

   For $n \ge 1$ $W(n) = I$ if there exists a move getting you to a II-spot, but is I otherwise. More formally if you can remove $\{a_1 < \cdots < a_k\}$ from the board then

   $W(n) = II$ if $0 \le n \le a_1 - 1$.

   For $n \ge a_1$

   - $W(n) = I$ if there exists a_i such that $n - a_i \ge 0$ and $W(n-a_i) = II$
   - $W(n) = II$ otherwise.

   For $NIM(1,2)$ here is table of who wins

n	0	1	2	3	4	5	6
W	II	I	I	II	I	I	II

   AH-HA: I wins iff $n \equiv 1,2 \pmod 3$.

   For $NIM(1,3)$ here is table of who wins

n	0	1	2	3	4	5	6
W	II	I	II	I	II	I	II

   AH-HA: I wins iff $n \equiv 1 \pmod 2$.

For $NIM(1,4)$ here is table of who wins

n	0	1	2	3	4	5	6	7	8	9
W	II	I	II	I	I	II	I	II	I	I

AH-HA: I wins iff $n \equiv 1,3,4 \pmod 5$.

For $NIM(1,5)$ here is table of who wins

n	0	1	2	3	4	5	6
W	II	I	II	I	II	I	II

AH-HA: I wins iff $n \equiv 1 \pmod 2$.

For $NIM(1,6)$ here is table of who wins

n	0	1	2	3	4	5	6	7	8	9	10	11	12	13
W	II	I	II	I	II	I	I	II	I	II	I	II	I	I

AH-HA: I wins iff $n \equiv 1,3,5,6 \pmod 7$.

Is there a pattern in all of this? YES

If $L \equiv 0 \pmod 2$ then
I wins iff $n \equiv 1,3,\ldots,L-1 \pmod{L+1}$ OR $n \equiv L \pmod{L+1}$

If $L \equiv 1 \pmod 2$ then

$$I \text{ wins iff } n \equiv 1 \pmod 2$$

END OF SOLUTION TO PROBLEM TWO

3. (40 points) Read my notes on Mono Squares which is posted. You may use Lemma 2.4 from those notes.

   (a) Show that there exists a number M such that for all 3-colorings of the $M \times M$ grid there is a mono square.

   (b) Show that for all c there exists a number M_c such that for all c-colorings of the $M_c \times M_c$ grid there is a mono square.

FOR SOLUTION SEE POSTED NOTES ON MONO SQUARES

4. (30 points) Let $A = \{1, 2, 3, 4\}$.

   (a) How many relations are there over the set A?

   (b) Of those, how many are functions?

   (c) If I pick a relation at random what is the probability that its a function?

SOLUTION TO PROBLEM FOUR

A relation of A is a subset of $A \times A$.

a) $A \times A$ is of size 16. So there are 2^{16} relations over A.

b) To count the number of functions we count:

How many numbers can 1 map to: 4

How many numbers can 2 map to: 4

How many numbers can 3 map to: 4

How many numbers can 4 map to: 4

So the answer is 4^4.

c) The probability that you pick a function is
$\frac{4^4}{2^{16}} = \frac{2^8}{2^{16}} = \frac{1}{2^8} \sim 0.004$.
JUST CURIOUS: What if the $A = \{1, 2, 3\}$. Then the answer would be
$\frac{3^3}{2^9} = \frac{27}{512} \sim 0.053$.
JUST CURIOUS: What if the $A = \{1, 2\}$. Then the answer would be
$\frac{2^2}{2^4} = \frac{1}{4} = 0.25$.

END OF SOLUTION TO PROBLEM 4