

## 250, Midterm I

**Do not open this exam until you are told. Read these instructions:**

1. This is a closed book exam. **No calculators, notes, or other aids are allowed.** If you have a question during the exam, please raise your hand.
2. **You must turn in your exam immediately when time is called at the end.**
3. 6 problems which add up to 98 points. You get 2 points for getting your NAME, SIGNATURE, SID, SECTION NUMBER correct on this page which we consider problem 0. 100 points total. 1 hour 30 minutes. Each question's point value is indicated.
4. In order to be eligible for as much partial credit as possible, show all of your work for each problem, **write legibly**, and **clearly indicate** your answers. Credit **cannot** be given for illegible answers.
5. After the last page there is paper for scratch work. If you need extra scratch paper **after** you have filled these areas up, please raise your hand. Scratch paper must be turned in with your exam, with your name and ID number written on it, but scratch paper **will not** be graded.
6. Please write out the following statement: "*I pledge on my honor that I will not give or receive any unauthorized assistance on this examination.*"

7. Fill in the following:

NAME :  
SIGNATURE :  
SID :  
SECTION NUMBER :

## SCORES ON PROBLEMS

|         |
|---------|
| Prob 0: |
| Prob 1: |
| Prob 2: |
| Prob 3: |
| Prob 4: |
| Prob 5: |
| Prob 6: |
| TOTAL   |

1. (18 points) For each of the following fill in the blank. No proofs required.

(a) Give a Boolean Formula on the five variables  $x_1, x_2, x_3, x_4, x_5$  that has exactly two satisfying assignments.

(b) Give the contrapositive of the following statement:

*If you do not hand in all of the homework and you do not take all of the quizzes then you will get an F in the course.*

- (c) Give a Boolean Formula on the three variables  $x_1, x_2, x_3$  that has exactly zero satisfying assignments and in which each variable appears at least once and at most twice.

2. (15 points)

- (a) Complete the truth table for the following function: on input  $b_2b_1b_0$ , a 3 bit number, output 1 if the number is a prime, and output 0 if the number is not a prime. (Note that 0 and 1 are NOT primes).

| $b_2$ | $b_1$ | $b_0$ |  |
|-------|-------|-------|--|
| 0     | 0     | 0     |  |
| 0     | 0     | 1     |  |
| 0     | 1     | 0     |  |
| 0     | 1     | 1     |  |
| 1     | 0     | 0     |  |
| 1     | 0     | 1     |  |
| 1     | 1     | 0     |  |
| 1     | 1     | 1     |  |

- (b) Use the method given in class to obtain a Boolean Formula from this truth table. DO NOT SIMPLIFY.

- (c) Draw a circuit for the formula you gave in part 2b. DO NOT SIMPLIFY. You may use AND, OR, and NOT gates only. The gates may have any number of inputs and any number of outputs.

3. (20 points). In this problem all domains are subsets of the real numbers. Consider the following sentence:

$$(\exists x_1)(\forall x_2)[x_1 \leq x_2] \wedge (\exists y_1)(\forall y_2)[y_2 \leq y_1].$$

- (a) Give an infinite domain  $D$  such that if the quantifiers range over  $D$  then the sentence is TRUE.
- (b) Give an infinite domain  $D$  such that if the quantifiers range over  $D$  then the sentence is FALSE.
- (c) Give a finite nonempty domain  $D$  such that if the quantifiers range over  $D$  then the sentence is TRUE.

4. (10 points) A number is GASARCHIAN if it is of the form  $a + b\sqrt{2} + c\sqrt{3}$  where  $a, b, c$  are **natural numbers**. For each of the following say if it is TRUE or FALSE and briefly explain why.
- (a) If  $x$  and  $y$  are GASARCHIAN then  $x + y$  is GASARCHIAN.
  - (b) If  $x$  and  $y$  are GASARCHIAN then  $x - y$  is GASARCHIAN.

## Scratch Work