1. (40 points) Throughout this problem Bill has a 2-sided dice with numbers 1, 2 and a 3-sided die with numbers 1, 2, 3.

(a) (15 points) Assume both dice are fair. Bill throws both of them. For $2 \leq i \leq 5$ give the prob that the sum is $i$.

(b) (20 points) Let $0 \leq p \leq \frac{1}{2}$. Assume the 2-sided dice is fair but the 3-sided dice has

- Prob of 1 = $p$
- Prob of 2 = $1 - 2p$
- Prob of 3 = $p$

Bill throws both of them. For $2 \leq i \leq 5$ give the prob that the sum is $i$.

(c) (5 points) Let $p$ be as in the last part. Is there a value of $p$ such that all of the sums 2, 3, 4, 5 come up with the same probability.

(d) (0 points but thing about it) Can you load two 6-sided dice to get fair sums?

SOLUTION TO PROBLEM ONE

1) Both dice are fair.

Prob(2) = Prob of (1, 1) = $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$.

Prob(3) = Prob or either (1, 2) or (2, 1) = $2 \times \frac{1}{2} \times \frac{1}{3} = \frac{1}{3}$.

Prob(4) = Prob or either (1, 3) or (2, 2) = $2 \times \frac{1}{2} \times \frac{1}{3} = \frac{1}{3}$.

Prob(5) = Prob or (2, 3) = $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$.

2)

Prob(2) = Prob of (1, 1) = $\frac{1}{2} \times p = \frac{p}{2}$.

Prob(3) = Prob or either (1, 2) or (2, 1) = $\frac{1}{2} \times (1 - 2p) + \frac{1}{2}p = \frac{1-p}{2}$.

Prob(4) = Prob or either (1, 3) or (2, 2) = $\frac{1}{2} \times p + \frac{1}{2} \times (1 - 2p) = \frac{1-p}{2}$.

Prob(5) = Prob or (2, 3) = $\frac{1}{2} \times p = \frac{p}{2}$.

3) If $p = 1/2$ then all of the probabilities are $\frac{1}{4}$.  

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2. (60 points) On the planet Vorlon they play a game that is similar to what we call Poker but with a different deck of cards.

Every card has a rank from \{1, 2, \ldots, 7\}.
Every card has a suite from \{R, B\}.
Every player gets 3 cards.
In most of the questions we will ask for the prob of a certain type of hand. Give the answer to 4 places since the last question is to rank them.

(a) What is prob of a straight that is NOT a flush (e.g., 3R, 4R, 5B) We DO allow wrap-around, so 7-1-2 counts.

(b) What is prob of a flush that is NOT a straight (e.g., 2R, 4R, 9R)

(c) What is prob of a straight flush (e.g., 3R, 4R, 6R) We DO allow wrap-around, so 7-1-2 counts.

(d) What is prob of a pair (e.g., 3R, 3B, 7R). Note that a pair cannot be a straight of a flush.

(e) What is prob of getting NOTHING- a hand that is neither a straight, nor a flush, nor does it contain 2 of a kind. (e.g., 3R, 5R, 6B)

(f) Rank the types of hands from most likely to least likely.

**SOLUTION TO PROBLEM TWO**

Note that the total number of hands is \( \binom{14}{3} = 364 \).

(a) A straight that is NOT a flush.

Pick a rank \( r \) — there are 7 ways to do this. Then you have \( r, r + 1, r + 2 \). Now pick for each card R or B, but DO NOT pick RRR or BBB so you pick one of 6 R-B sequences. So 42. So prob is \( \frac{42}{364} = \frac{3}{26} \approx 0.12 \). NOTE FOR LATER: 42 ways to get a straight, NOT a flush.

(b) A flush that is NOT a straight (e.g., 2R, 4R, 9R)

Pick a suit — there are 2 ways to do this. Then pick 3 ranks — there are \( \binom{7}{3} \) ways to do that. NO- need to make sure they are
not a straight. There are 7 straights: 123, 234, ..., 712. So prob is 
\[2 \times \left(\binom{7}{3} - 7\right) = 2\left(35 - 7\right) = 2 \times 28 = 56.\] So Prob is 
\[\frac{56}{364} \sim 0.154.\] NOTE FOR LATER: 56 ways to get a flush, NOT a 
straight.

(c) A straight flush.

Pick a rank — there are 7 ways to do this. Pick a suite — there are 2 ways to do this. So there are \(7 \times 2 = 14\) ways to get a straight flush. So prob is \(\frac{14}{364} \sim 0.038.\)

NOTE FOR LATER: 14 ways to get a straight flush.

(d) A pair.

Pick a rank — there are 7 ways to do this. The suits are determined-one will be R and one will be B. Then pick the other card — there are \(14 - 2 = 12\) ways to do this. So there are \(7 \times 12 = 84\) ways to get a pair. So prob is \(\frac{84}{364} \sim 0.23.\)

NOTE FOR LATER: 84 ways to get a pair.

(e) NOTHING.

All of the above types are disjoint. Hence we need only subtract. The number of hands with NOTHING is 
\[364 - 42 - 56 - 14 - 84 = 168.\] So the prob of getting nothing is \(\frac{168}{364} \sim 0.46.\)

(f) RANK: from most likely to least likely:

NOTHING: Prob \(\sim 0.46.\)

A PAIR: Prob \(\sim 0.23.\)

FLUSH THAT IS NOT A STRAIGHT: Prob \(\sim 0.154.\)

STRAIGHT THAT IS NOT A FLUSH: Prob \(\sim 0.12.\)

STRAIGHT FLUSH: Prob \(\sim 0.038.\)