Homework 4 MORALLY Due March 07 at 9:00AM
WARNING: THIS HW IS SIX PAGES LONG!!!!!!!!!!!!!!!

1. (0 points but please DO IT) What is your name?

2. (20 points)
   (a) (20 points) Show that, for all primes \( p \), \( p^{1/5} \notin \mathbb{Q} \). USE the Unique Factorization.
   (b) (0 points) Try to do it using the MOD method. Discuss if this works or not.
3. (20 points) Let PRIMES be the set of primes.

Let $\text{MOD}_{i,j}$ be the set of numbers that are $\equiv i \pmod{j}$.
Let $\text{PRIMES}_{i,j} = \text{PRIMES} \cap \text{MOD}_{i,j}$.

For example
$\text{PRIMES}_{1,4}$ is the set of primes that are $\equiv 1 \pmod{4}$.

(a) Show that $\text{PRIMES}_{3,4}$ is infinite. (Hint: If $p_1, \ldots, p_L$ are all of the
primes $\equiv 3 \pmod{4}$ then look at $4p_1 \cdots p_L - 1$.)

(b) Show that $\text{PRIMES}_{5,6}$ is infinite.

(c) Give an infinite sequence $i_1 < j_1 < i_2 < j_2 < \cdots$ such that
$\text{PRIMES}_{i_1,j_1}$ is EMPTY.
$\text{PRIMES}_{i_2,j_2}$ is EMPTY.
$\text{PRIMES}_{i_3,j_3}$ is EMPTY.
etc.

(d) Give an infinite sequence $i_1 < j_1 < i_2 < j_2 < \cdots$ such that
$\text{PRIMES}_{i_1,j_1}$ is FINITE but not EMPTY.
$\text{PRIMES}_{i_2,j_2}$ is FINITE but not EMPTY.
$\text{PRIMES}_{i_3,j_3}$ is FINITE but not EMPTY.
etc.
4. (30 points- 10 points each) For each of the following sequences find a
simple function $A(n)$ such that the sequence is $A(1), A(2), \ldots$. (I am
not going to define simple rigorously, but just keep it simple.)

(a) $10, -17, 24, -31, 38, -45, 52, \ldots$
(b) $-1, 1, 5, 13, 29, 61, 125, \ldots$
(c) $6, 9, 14, 21, 30, 41, 54, \ldots$
5. (25 points) In Grand Fenwick they have two types of coins: one worth 100 cents, and one worth 101 cents.

(a) (15 points) Show that for all \(n \geq 9900\) one can form \(n\) with these two types of coins.

(In Math:

\[
(\forall n \geq 9900)(\exists x, y \in \mathbb{N})[n = 100x + 101y].
\]

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(b) (10 points) Prove or Disprove: There is NO way to form 9899 cents in Grand Fenwick.

(In Math:

\[
(\forall x, y \in \mathbb{N})[9899 \neq 100x + 101y].
\]

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(c) (0 points but you should do it) If you DID NOT KNOW the bound of 9900 how would you find it. HINT: Use Wolfram Alpha.
6. (Extra Credit) Prove or Disprove: there is a second order sentence that is TRUE of $Q + Q$ but false of $Q$. 