Combinatorial Identities

250H
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Proof (1): The number of subsets of \{1, 2, ... , n\} is $2^n$. From that set we can choose 0 elements or 1 elements or ... or $n$ elements. Thus, $2^n = \binom{n}{0} + \binom{n}{1} + \ldots + \binom{n}{n}$. ✸
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Proof (2): Consider the identity, \((x + y)^n = \sum\binom{n}{i} x^i y^{n-i}\)

Choose \( x = y = 1 \). Now we have \((1 + 1)^n = \sum\binom{n}{i} 1^i 1^{n-i}\) or \(2^n = \sum\binom{n}{i}\).

Thus, \( 2^n = \binom{n}{0} + \binom{n}{1} + \ldots + \binom{n}{n} \). ★
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Option 2: We can pick 0 girls and $n$ boys, 1 girl and $n-1$ boys, ..., $n$ girls and $n-1$ boys.
So we have $\sum \binom{n}{i}^2$ ways.
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Option 2: We can pick 0 girls and $n$ boys, 1 girl and $n-1$ boys, ..., $n$ girls and $n-1$ boys.

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This is another identity: $\sum \binom{n}{i}^2 = \binom{2n}{n}$.
Combinatorial Identities

1. \((x + y)^n = \sum \binom{n}{i} x^i y^{n-i}\)

2. \(\sum \binom{n}{i}^2 = \binom{2n}{n}\)