Please Fill Out All of Your Courses Teaching Evals

May 10, 2022
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1) Teachers read their teaching eval comments for self-improvement.
2) The Teach Eval Chair reads others teaching evals and may discuss it with the teacher.
3) The Undergrad Chair reads others teaching evals and may discuss it with the teacher.
4) The committee that gives out teaching awards reads the teaching evals to help make a decision.
5) When a teacher goes up for tenure, the teaching evals are used in the teaching report.
6) The biggest problem we have for all of the above is when not that many students fill them out. Hence
7) Please Fill Out the Teaching Evals in All of your Courses
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7) **Please Fill Out the Teaching Evals in All of your Courses**
How to Use the Half Method

(1) Given Upper Bound
(2) NOT Given Upper Bound

May 10, 2022
How Your Program Shows $f(45, 26) \leq \frac{32}{78}$

May 10, 2022
Your Programs FC Step on $f(45, 26)$

$$f(m, s) \leq \max\left\{ \frac{1}{3}, \min\left\{ \frac{m}{s} \times \frac{1}{\lfloor 2m/s \rfloor}, 1 - \frac{m}{2} \times \frac{1}{\lceil 2m/s \rceil} \right\}\right\}.$$
Your Programs FC Step on \( f(45, 26) \)

\[
f(m, s) \leq \max \left\{ \frac{1}{3}, \min \left\{ \frac{m}{s} \times \frac{1}{\lfloor 2m/s \rfloor}, 1 - \frac{m}{2} \times \frac{1}{\lfloor 2m/s \rfloor} \right\} \right\}.
\]

\[
\frac{2m}{s} = \frac{90}{26} = \frac{45}{13} \sim 3.46 \quad \lfloor \frac{2m}{s} \rfloor = 3 \quad \lceil \frac{2m}{s} \rceil = 4
\]
Your Programs FC Step on $f(45, 26)$

\[
f(m, s) \leq \max\left\{ \frac{1}{3}, \min\left\{ \frac{m}{s} \times \frac{1}{\left\lceil 2m/s \right\rceil}, 1 - \frac{m}{2} \times \frac{1}{\left\lfloor 2m/s \right\rfloor} \right\} \right\}.
\]

\[
\frac{2m}{s} = \frac{90}{26} = \frac{45}{13} \approx 3.46 \quad \left\lceil \frac{2m}{s} \right\rceil = 3 \quad \left\lfloor \frac{2m}{s} \right\rfloor = 4
\]

\[
f(45, 26) \leq \max\left\{ \frac{1}{3}, \min\left\{ \frac{45}{26} \times \frac{1}{4}, 1 - \frac{45}{26} \times \frac{1}{3} \right\} \right\} = \min\left\{ \frac{45}{104}, 1 - \frac{15}{26} \right\}
\]
Your Programs FC Step on $f(45, 26)$

\[ f(m, s) \leq \max \left\{ \frac{1}{3}, \min \left\{ \frac{m}{s} \times \frac{1}{\lfloor 2m/s \rfloor}, 1 - \frac{m}{2} \times \frac{1}{\lfloor 2m/s \rfloor} \right\} \right\}. \]

\[ \frac{2m}{s} = \frac{90}{26} = \frac{45}{13} \sim 3.46 \quad \lfloor \frac{2m}{s} \rfloor = 3 \quad \lceil \frac{2m}{s} \rceil = 4 \]

\[ f(45, 26) \leq \max \left\{ \frac{1}{3}, \min \left\{ \frac{45}{26} \times \frac{1}{4}, 1 - \frac{45}{26} \times \frac{1}{3} \right\} \right\} = \min \left\{ \frac{45}{104}, 1 - \frac{15}{26} \right\} \]

\[ = \min \left\{ \frac{45}{104}, \frac{11}{26} \right\} = \frac{11}{26} \sim 0.423 \]

We want $f(m, s) \leq \frac{32}{78} \sim 0.410$. So FC NOT powerful enough.
The $V$-step

Your program will try $V = 2, 3, 4, \ldots$ and find out that

$\exists$ a share $\geq \frac{45}{26} \times 1 = \frac{45}{52}$.

$\exists$ a share $\leq \frac{45}{26} \times \frac{1}{5} = \frac{9}{26} < \frac{32}{78}$.
The $V$-step

Your program will try $V = 2, 3, 4, \ldots$ and find out that

If someone gets $\leq 2$ shares then $\exists$ a share $\geq \frac{45}{26} \times \frac{1}{2} = \frac{45}{52}$.
The V-step

Your program will try $V = 2, 3, 4, \ldots$ and find out that

If someone gets $\leq 2$ shares then $\exists$ a share $\geq \frac{45}{26} \times \frac{1}{2} = \frac{45}{52}$.

Buddy is $\leq 1 - \frac{45}{52} = \frac{7}{52} < \frac{32}{78}$. 

Some students get 3 shares.

Some students get 4 shares.
The $V$-step

Your program will try $V = 2, 3, 4, \ldots$ and find out that

If someone gets $\leq 2$ shares then $\exists$ a share $\geq \frac{45}{26} \times \frac{1}{2} = \frac{45}{52}$.

Buddy is $\leq 1 - \frac{45}{52} = \frac{7}{52} < \frac{32}{78}$.

If someone gets $\geq 5$ shares then $\exists$ a share $\leq \frac{45}{26} \times \frac{1}{5} = \frac{9}{26} < \frac{32}{78}$.
The $V$-step

Your program will try $V = 2, 3, 4, \ldots$ and find out that

If someone gets $\leq 2$ shares then $\exists$ a share $\geq \frac{45}{26} \times \frac{1}{2} = \frac{45}{52}$. Buddy is $\leq 1 - \frac{45}{52} = \frac{7}{52} < \frac{32}{78}$.

If someone gets $\geq 5$ shares then $\exists$ a share $\leq \frac{45}{26} \times \frac{1}{5} = \frac{9}{26} < \frac{32}{78}$.

$V = 4$. 
The $V$-step

Your program will try $V = 2, 3, 4, \ldots$ and find out that

If someone gets $\leq 2$ shares then $\exists$ a share $\geq \frac{45}{26} \times \frac{1}{2} = \frac{45}{52}$. 
Buddy is $\leq 1 - \frac{45}{52} = \frac{7}{52} < \frac{32}{78}$.

If someone gets $\geq 5$ shares then $\exists$ a share $\leq \frac{45}{26} \times \frac{1}{5} = \frac{9}{26} < \frac{32}{78}$.

$V = 4$.

Some students gets 3 shares.
Some students gets 4 shares.
Equations Step

\[
3s_3 + 4s_4 = 90 \\
s_3 + s_4 = 26
\]
Equations Step

\[3s_3 + 4s_4 = 90\]
\[s_3 + s_4 = 26\]

\[s_3 = 14\]
\[s_4 = 12.\]
Equations Step

\[3s_3 + 4s_4 = 90\]
\[s_3 + s_4 = 26\]

\[s_3 = 14\]
\[s_4 = 12.\]

14 students get 3 shares

Note: One way for HALF to not work is if these equations have a solution that is not in \(\mathbb{N}\).
Equations Step

\[3s_3 + 4s_4 = 90\]
\[s_3 + s_4 = 26\]

\[s_3 = 14\]
\[s_4 = 12.\]

14 students get 3 shares
12 students get 4 shares
Equations Step

\[ 3s_3 + 4s_4 = 90 \]
\[ s_3 + s_4 = 26 \]

\[ s_3 = 14 \]
\[ s_4 = 12. \]

14 students get 3 shares
12 students get 4 shares
Note:
there are \( 3 \times 14 = 42 \) 3-shares
there are \( 4 \times 12 = 48 \) 4-shares.
Equations Step

\[ 3s_3 + 4s_4 = 90 \]
\[ s_3 + s_4 = 26 \]

\[ s_3 = 14 \]
\[ s_4 = 12. \]

14 students get 3 shares
12 students get 4 shares
Note:
there are \( 3 \times 14 = 42 \) 3-shares
there are \( 4 \times 12 = 48 \) 4-shares.

**Note** One way for HALF to not work is if these equations have a solution that is not in \( \mathbb{N} \).
The $\beta$ Step

\[
\begin{pmatrix}
\frac{32}{78} & 48 \text{ 4-shs} & \frac{46}{78} \\
\beta & 0 & \gamma \\
\frac{42}{78} & 3\text{-shs} & \frac{46}{78}
\end{pmatrix}
\]
The $\beta$ Step

$$\left( \begin{array}{c} 48 \text{ 4-shs} \\ \frac{32}{78} \end{array} \right) \left[ \begin{array}{c} 0 \\ \beta \end{array} \right] \left( \begin{array}{c} 42 \text{ 3-shs} \\ \frac{46}{78} \end{array} \right)$$

**Want** if $\exists$ 4-share $\geq \beta$ then some piece $\leq \frac{32}{78}$. 
The $\beta$ Step

\[
\begin{pmatrix}
32/78 & 48 \text{ 4-shs} \\
\beta & 0 \\
\gamma & 42 \text{ 3-shs}
\end{pmatrix}
\]

Want if $\exists$ 4-share $\geq \beta$ then some piece $\leq \frac{32}{78}$.

Alice has $p_1 \leq p_2 \leq p_3 \leq p_4$. 
The $\beta$ Step

$$\left( \begin{array}{c} 32 \\ 78 \end{array} \right) \left[ \begin{array}{c} 48 \text{ 4-shs} \\ 0 \end{array} \right] \left( \begin{array}{c} 42 \text{ 3-shs} \\ \beta \end{array} \right) \left( \begin{array}{c} \beta \\ \gamma \end{array} \right) \left( \begin{array}{c} 46 \\ 78 \end{array} \right)$$

**Want** if $\exists$ 4-share $\geq \beta$ then some piece $\leq \frac{32}{78}$.

Alice has $p_1 \leq p_2 \leq p_3 \leq p_4$.
Assume $p_4 \geq \beta$ and later pick $\beta$ to get a contradiction.

$$p_1 + p_2 + p_3 + p_4 = \frac{45}{26}$$

$$p_1 + p_2 + p_3 = \frac{45}{26} - p_4 \leq \frac{45}{26} - \beta$$
The $\beta$ Step

$$
\begin{pmatrix}
32/78 & 48 \text{ 4-shs} \\
46/78 & 0 \\
42 \text{ 3-shs} & \beta \\
\gamma & 46/78
\end{pmatrix}
$$

Want if $\exists$ 4-share $\geq \beta$ then some piece $\leq 32/78$.

Alice has $p_1 \leq p_2 \leq p_3 \leq p_4$.

Assume $p_4 \geq \beta$ and later pick $\beta$ to get a contradiction.

$p_1 + p_2 + p_3 + p_4 = 45/26$

$p_1 + p_2 + p_3 = 45/26 - p_4 \leq 45/26 - \beta$

$p_1 \leq \frac{1}{3}(45/26 - \beta)$. Want $\beta$ so that $p_1 \leq 32/78$:
The $\beta$ Step

\[
\begin{pmatrix}
    \frac{48}{78} & 48 	ext{-shs} \\
    \frac{32}{78} & \beta
\end{pmatrix}
\begin{pmatrix}
    0 \\
    \gamma
\end{pmatrix}
\begin{pmatrix}
    \frac{42}{78} & 32 	ext{-shs} \\
    \frac{46}{78}
\end{pmatrix}
\]

**Want** if $\exists$ 4-share $\geq \beta$ then some piece $\leq \frac{32}{78}$.

Alice has $p_1 \leq p_2 \leq p_3 \leq p_4$.
Assume $p_4 \geq \beta$ and later pick $\beta$ to get a contradiction.

\[
p_1 + p_2 + p_3 + p_4 = \frac{45}{26}
\]

\[
p_1 + p_2 + p_3 = \frac{45}{26} - p_4 \leq \frac{45}{26} - \beta
\]

\[
p_1 \leq \frac{1}{3}(\frac{45}{26} - \beta).
\]

Want $\beta$ so that $p_1 \leq \frac{32}{78}$:

\[
\frac{1}{3}(\frac{45}{26} - \beta) \leq \frac{32}{78}
\]
The $\beta$ Step

$$\begin{pmatrix} 48 \mathrm{ 4-shs} & 0 \\ 32 \beta \gamma & 42 \mathrm{ 3-shs} \end{pmatrix} \begin{pmatrix} \frac{32}{78} \\ \beta \gamma \frac{46}{78} \end{pmatrix}$$

Want if $\exists$ 4-share $\geq \beta$ then some piece $\leq \frac{32}{78}$.

Alice has $p_1 \leq p_2 \leq p_3 \leq p_4$.

Assume $p_4 \geq \beta$ and later pick $\beta$ to get a contradiction.

$p_1 + p_2 + p_3 + p_4 = \frac{45}{26}$

$p_1 + p_2 + p_3 = \frac{45}{26} - p_4 \leq \frac{45}{26} - \beta$

$p_1 \leq \frac{1}{3}(\frac{45}{26} - \beta)$. Want $\beta$ so that $p_1 \leq \frac{32}{78}$:

$\frac{1}{3}(\frac{45}{26} - \beta) \leq \frac{32}{78}$

$\beta \geq \frac{39}{78} = \frac{1}{2}$. Take $\beta = \frac{39}{78}$. 
The $\gamma$ Step (What you Really Want to See)

\[
\left( \begin{array}{c}
\frac{32}{78} \\
\beta \\
\gamma \\
\frac{46}{78}
\end{array} \right) [0] \left( \begin{array}{c}
48 \text{ 4-shs} \\
0 \\
42 \text{ 3-shs} \\
\frac{46}{78}
\end{array} \right)
\]
The $\gamma$ Step (What you Really Want to See)

\[
\begin{pmatrix}
\frac{32}{78} & 48 	ext{ 4-shs} & 0 \\
\beta & \gamma & \frac{42}{78} \\
\frac{46}{78} & 42 	ext{ 3-shs} & 0
\end{pmatrix}
\]

Want if $\exists$ 3-share $\leq \gamma$ then some piece $\leq \frac{32}{78}$. 
The $\gamma$ Step (What you Really Want to See)

\[
\begin{pmatrix}
32/78 & 48 \text{ 4-shs} & 0 & 42 \text{ 3-shs} & 46/78
\end{pmatrix}
\]

**Want** if $\exists$ 3-share $\leq \gamma$ then some piece $\leq 32/78$.

Bob has $p_1 \leq p_2 \leq p_3$. 

The $\gamma$ Step (What you Really Want to See)

$$
\begin{pmatrix}
\frac{32}{78} & \beta \\
\frac{78}{78} & \gamma
\end{pmatrix}
\begin{pmatrix}
48 & 4-\text{shs} \\
42 & 3-\text{shs}
\end{pmatrix}
= 
\begin{pmatrix}
0 \\
\frac{46}{78}
\end{pmatrix}
$$

**Want** if $\exists$ 3-share $\leq \gamma$ then some piece $\leq \frac{32}{78}$.

Bob has $p_1 \leq p_2 \leq p_3$.

Assume $p_1 \leq \gamma$ and later pick $\gamma$ to get a contradiction.

$p_1 + p_2 + p_3 = \frac{45}{26}$

$p_2 + p_3 = \frac{45}{26} - p_1 \geq \frac{45}{26} - \gamma$
**The $\gamma$ Step (What you Really Want to See)**

\[
\begin{pmatrix}
32/78 & 48 \text{ 4-shs} & 0 & 42 \text{ 3-shs} & 46/78 \\
\beta & \gamma
\end{pmatrix}
\]

**Want** if $\exists$ 3-share $\leq \gamma$ then some piece $\leq \frac{32}{78}$.

Bob has $p_1 \leq p_2 \leq p_3$.

Assume $p_1 \leq \gamma$ and later pick $\gamma$ to get a contradiction.

\[
p_1 + p_2 + p_3 = \frac{45}{26}
\]

\[
p_2 + p_3 = \frac{45}{26} - p_1 \geq \frac{45}{26} - \gamma
\]

\[
p_3 \geq \frac{1}{2}(\frac{45}{26} - \gamma). \textbf{Key} \text{ Look at buddy of } p_3.
\]
The $\gamma$ Step (What you Really Want to See)

\[
\begin{pmatrix}
48 & 4\text{-shs} \\
32/78 & \\
\beta & \gamma \\
42 & 3\text{-shs} \\
46/78 & 
\end{pmatrix}
\]

**Want** if $\exists$ 3-share $\leq \gamma$ then some piece $\leq \frac{32}{78}$.

Bob has $p_1 \leq p_2 \leq p_3$.

Assume $p_1 \leq \gamma$ and later pick $\gamma$ to get a contradiction.

\[
p_1 + p_2 + p_3 = \frac{45}{26}
\]

\[
p_2 + p_3 = \frac{45}{26} - p_1 \geq \frac{45}{26} - \gamma
\]

\[
p_3 \geq \frac{1}{2}(\frac{45}{26} - \gamma). \textbf{Key} \text{ Look at buddy of } p_3.
\]

\[
1 - p_3 \leq 1 - \frac{1}{2}(\frac{45}{26} - \gamma). \text{ Want } \gamma \text{ so that } 1 - p_3 \leq \frac{32}{78}:
\]
The $\gamma$ Step (What you Really Want to See)

$$\begin{pmatrix} 48 & \text{4-shs} \\ 32 & \beta \\ 78 & \gamma \\ 42 & \text{3-shs} \\ 46 & 78 \end{pmatrix}$$

Want if $\exists$ 3-share $\leq \gamma$ then some piece $\leq \frac{32}{78}$.

Bob has $p_1 \leq p_2 \leq p_3$.

Assume $p_1 \leq \gamma$ and later pick $\gamma$ to get a contradiction.

$p_1 + p_2 + p_3 = \frac{45}{26}$
$p_2 + p_3 = \frac{45}{26} - p_1 \geq \frac{45}{26} - \gamma$
$p_3 \geq \frac{1}{2}(\frac{45}{26} - \gamma)$. **Key** Look at buddy of $p_3$.

$1 - p_3 \leq 1 - \frac{1}{2}(\frac{45}{26} - \gamma)$. Want $\gamma$ so that $1 - p_3 \leq \frac{32}{78}$:

$1 - \frac{1}{2}(\frac{45}{26} - \gamma) \leq \frac{32}{78}$. 
The $\gamma$ Step (What you Really Want to See)

\[
\begin{pmatrix}
32/78 & 48 - \text{4-shs} & 0 \\
\beta & \gamma & 42 - \text{3-shs}
\end{pmatrix}
\]

**Want** if $\exists$ 3-share $\leq \gamma$ then some piece $\leq \frac{32}{78}$.

Bob has $p_1 \leq p_2 \leq p_3$.

Assume $p_1 \leq \gamma$ and later pick $\gamma$ to get a contradiction.

\[p_1 + p_2 + p_3 = \frac{45}{26}\]
\[p_2 + p_3 = \frac{45}{26} - p_1 \geq \frac{45}{26} - \gamma\]
\[p_3 \geq \frac{1}{2}(\frac{45}{26} - \gamma). \textbf{Key} \text{ Look at buddy of } p_3.\]

\[1 - p_3 \leq 1 - \frac{1}{2}(\frac{45}{26} - \gamma). \text{ Want } \gamma \text{ so that } 1 - p_3 \leq \frac{32}{78}:\]
\[1 - \frac{1}{2}(\frac{45}{26} - \gamma) \leq \frac{32}{78}.
\[\gamma \leq \frac{43}{78}. \text{ Take } \gamma = \frac{43}{78}.\]
There are 42 3-shares. 
There are 48 4-shares 

\[ \beta = \frac{1}{2} \leq \frac{1}{2} \leq \gamma = \frac{43}{78} \]

42 ≠ 48 so SUCCESS!
VHALF Step

There are 42 3-shares.
There are 48 4-shares

$$\beta = \frac{1}{2} \leq \frac{1}{2} \leq \gamma = \frac{43}{78}$$

42 $\neq$ 48 so SUCCESS!

**Note** One way for the HALF method to fail is if $\gamma < \beta$
How Your Program Finds the Answer

May 10, 2022
V-Step and Equation Step

\[ V = \lceil 2m \rceil = \lceil 9026 \rceil = 4. \]

\[ s_3 = 14 \]

\[ s_4 = 12. \]
V-Step and Equation Step

\[ V = \left\lceil \frac{2m}{s} \right\rceil = \left\lceil \frac{90}{26} \right\rceil = 4. \]
V-Step and Equation Step

\[ V = \left\lceil \frac{2m}{s} \right\rceil = \left\lceil \frac{90}{26} \right\rceil = 4. \]

\[ s_3 = 14 \]
V-Step and Equation Step

\[ V = \left\lceil \frac{2m}{s} \right\rceil = \left\lceil \frac{90}{26} \right\rceil = 4. \]
\[ s_3 = 14 \]
\[ s_4 = 12. \]
\[ \beta = \frac{1}{2} \text{-Step} \]

\[
\begin{pmatrix}
\alpha & 48 \text{ 4-shs} \\
\beta & 0 \\
\gamma & 42 \text{ 3-shs} \\
1 - \alpha & \\
\end{pmatrix}
\]
$\beta = \frac{1}{2}$-Step

\[
\begin{bmatrix}
\alpha & \beta & \gamma & 1 - \alpha \\
48 \text{ 4-shs} & 0 & 42 \text{ 3-shs}
\end{bmatrix}
\]

We want to set $\alpha$ so that $\beta = \frac{1}{2}$.

Alice has $p_1 \leq p_2 \leq p_3 \leq p_4$. 

Note: The equation $p_1 + p_2 + p_3 + p_4 = \frac{45}{26}$ seems to be a misinterpretation or a typo. It should be understood as the sum of shares rather than the equality of the shares.
We want to set $\alpha$ so that $\beta = \frac{1}{2}$.

Alice has $p_1 \leq p_2 \leq p_3 \leq p_4$.
Assume $p_4 \geq \beta = \frac{1}{2}$ and later pick $\alpha$ to get a contradiction.

$p_1 + p_2 + p_3 + p_4 = \frac{45}{26}$
$p_1 + p_2 + p_3 = \frac{45}{26} - p_4 \leq \frac{45}{26} - \frac{1}{2}$
\( \beta = \frac{1}{2} \)-Step

\[
\begin{pmatrix}
48 & \text{4-shs} \\
42 & \text{3-shs}
\end{pmatrix}
\begin{bmatrix}
\alpha & 0 & \beta & \gamma & 1 - \alpha
\end{bmatrix}
\]

We want to set \( \alpha \) so that \( \beta = \frac{1}{2} \).

Alice has \( p_1 \leq p_2 \leq p_3 \leq p_4 \).

Assume \( p_4 \geq \beta = \frac{1}{2} \) and later pick \( \alpha \) to get a contradiction.

\[
p_1 + p_2 + p_3 + p_4 = \frac{45}{26}
\]

\[
p_1 + p_2 + p_3 = \frac{45}{26} - p_4 \leq \frac{45}{26} - \frac{1}{2}
\]

\[
p_1 \leq \frac{1}{3} \left( \frac{45}{26} - \frac{1}{2} \right). \text{ Want } \alpha \text{ so that } p_1 \leq \alpha:
\]
\( \beta = \frac{1}{2} \)-Step

\[
\begin{array}{cccc}
( & 48 \text{ 4-shs} & )[ & 0 ]( & 42 \text{ 3-shs} & ) \\
\alpha & \beta & \gamma & 1 - \alpha
\end{array}
\]

We want to set \( \alpha \) so that \( \beta = \frac{1}{2} \).

Alice has \( p_1 \leq p_2 \leq p_3 \leq p_4 \).
Assume \( p_4 \geq \beta = \frac{1}{2} \) and later pick \( \alpha \) to get a contradiction.

\[
p_1 + p_2 + p_3 + p_4 = \frac{45}{26}
\]

\[
p_1 + p_2 + p_3 = \frac{45}{26} - p_4 \leq \frac{45}{26} - \frac{1}{2}
\]

\[
p_1 \leq \frac{1}{3} \left( \frac{45}{26} - \frac{1}{2} \right). \text{ Want } \alpha \text{ so that } p_1 \leq \alpha:
\]

\[
\frac{1}{3} \left( \frac{45}{26} - \frac{1}{2} \right) = \frac{32}{78}.
\]

We note that there are 48 > 45 4-shares that are all \leq \frac{1}{2}.
So we have \( \frac{32}{78} \) is an upper bound.
The $\gamma = \frac{1}{2}$ Step (What you Really Want to See)

$$
\begin{align*}
( & 48 \text{ 4-shs} & 0 & ) ( & 42 \text{ 3-shs} & ) \\
\alpha & & \beta & & \gamma & & 1 - \alpha
\end{align*}
$$
The $\gamma = \frac{1}{2}$ Step (What you Really Want to See)

$$\begin{pmatrix} 48 \text{ 4-shs} & | & 0 & | & 42 \text{ 3-shs} \\ \alpha & | & \beta & | & \gamma & | & 1 - \alpha \end{pmatrix}$$

**Want** if $\exists$ 3-share $\leq \gamma = \frac{1}{2}$ then some piece $\leq \alpha$. 
The $\gamma = \frac{1}{2}$ Step (What you Really Want to See)

\[
\begin{pmatrix}
\alpha & \beta & \gamma & 1 - \alpha \\
48 \text{ 4-shs} & 0 & 42 \text{ 3-shs}
\end{pmatrix}
\]

**Want** if $\exists$ 3-share $\leq \gamma = \frac{1}{2}$ then some piece $\leq \alpha$.

Bob has $p_1 \leq p_2 \leq p_3$. 
The $\gamma = \frac{1}{2}$ Step (What you Really Want to See)

$$
\begin{align*}
( & 48 \text{ 4-shs} ) & [ & 0 & ] ( & 42 \text{ 3-shs} & ) \\
\alpha & & \beta & & \gamma & & 1 - \alpha
\end{align*}
$$

**Want** if $\exists 3\text{-share} \leq \gamma = \frac{1}{2}$ then some piece $\leq \alpha$.

Bob has $p_1 \leq p_2 \leq p_3$.
Assume $p_1 \leq \frac{1}{2}$ and later pick $\alpha$ to get a contradiction.

$$
p_1 + p_2 + p_3 = \frac{45}{26}
$$

$$
p_2 + p_3 = \frac{45}{26} - \frac{1}{2}
$$
The $\gamma = \frac{1}{2}$ Step (What you Really Want to See)

\[
\begin{array}{cccc}
\alpha & \beta & \gamma & 1 - \alpha \\
(48 \text{ 4-shs}) & [0] & (42 \text{ 3-shs}) & \\
\end{array}
\]

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\]
\[
p_3 \geq \frac{1}{2}(\frac{45}{26} - \frac{1}{2}). \textbf{Key} \text{ Look at buddy of } p_3.
\]
The $\gamma = \frac{1}{2}$ Step (What you Really Want to See)

\[
\begin{pmatrix}
48 & 4\text{-shs} \\
\alpha
\end{pmatrix}
\begin{pmatrix}
0 \\
\beta
\end{pmatrix}
\begin{pmatrix}
42 & 3\text{-shs} \\
\gamma
\end{pmatrix}
\begin{pmatrix}
1 - \alpha
\end{pmatrix}
\]

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\[p_1 + p_2 + p_3 = \frac{45}{26}\]
\[p_2 + p_3 = \frac{45}{26} - \frac{1}{2}\]
\[p_3 \geq \frac{1}{2} \left(\frac{45}{26} - \frac{1}{2}\right)\]. **Key** Look at buddy of $p_3$.

\[1 - p_3 \leq 1 - \frac{1}{2} \left(\frac{45}{26} - \frac{1}{2}\right)\]. Want $\alpha$ so that $1 - p_3 \leq \alpha$: 
The $\gamma = \frac{1}{2}$ Step (What you Really Want to See)

( 48 4-shs )[ 0 ]( 42 3-shs )

$\alpha$ $\beta$ $\gamma$ $1 - \alpha$

Want if $\exists$ 3-share $\leq \gamma = \frac{1}{2}$ then some piece $\leq \alpha$.

Bob has $p_1 \leq p_2 \leq p_3$.
Assume $p_1 \leq \frac{1}{2}$ and later pick $\alpha$ to get a contradiction.

$p_1 + p_2 + p_3 = \frac{45}{26}$
$p_2 + p_3 = \frac{45}{26} - \frac{1}{2}$
$p_3 \geq \frac{1}{2}(\frac{45}{26} - \frac{1}{2})$. Key Look at buddy of $p_3$.

$1 - p_3 \leq 1 - \frac{1}{2}(\frac{45}{26} - \frac{1}{2})$. Want $\alpha$ so that $1 - p_3 \leq \alpha$:

$1 - \frac{1}{2}(\frac{45}{26} - \frac{1}{2}) \leq \alpha$. 
The $\gamma = \frac{1}{2}$ Step (What you Really Want to See)

$$
\begin{array}{cccc}
\alpha & 48 \text{ 4-shs} & 0 & 42 \text{ 3-shs} \\
\beta & \gamma & 1 - \alpha \\
\end{array}
$$

**Want** if $\exists$ 3-share $\leq \gamma = \frac{1}{2}$ then some piece $\leq \alpha$.

Bob has $p_1 \leq p_2 \leq p_3$.
Assume $p_1 \leq \frac{1}{2}$ and later pick $\alpha$ to get a contradiction.

$p_1 + p_2 + p_3 = \frac{45}{26}$
$p_2 + p_3 = \frac{45}{26} - \frac{1}{2}$

$p_3 \geq \frac{1}{2} (\frac{45}{26} - \frac{1}{2})$. **Key** Look at buddy of $p_3$.

$1 - p_3 \leq 1 - \frac{1}{2} (\frac{45}{26} - \frac{1}{2})$. Want $\alpha$ so that $1 - p_3 \leq \alpha$:

$1 - \frac{1}{2} (\frac{45}{26} - \frac{1}{2}) \leq \alpha$.

$1 - \frac{1}{2} (\frac{45}{26} - \frac{1}{2}) = \frac{5}{13}$. 
Final Step

We get that both $\frac{32}{78}$ and $\frac{5}{13}$ are potential upper bounds.
Final Step

We get that both $\frac{32}{78}$ and $\frac{5}{13}$ are potential upper bounds.

Run $VHALF$ on both of them to find out that $\frac{5}{13}$ is not an upper bound.
Final Step

We get that both $\frac{32}{78}$ and $\frac{5}{13}$ are potential upper bounds.

Run $VHALF$ on both of them to find out that $\frac{5}{13}$ is not an upper bound.

So answer is $\frac{32}{78}$. 