SATisfiability
Satisfiability (SAT)

**Def** \( \phi(\vec{x}) \in SAT \) if there is \( \vec{b} \) such that \( \phi(\vec{b}) = T \). If \( \vec{b} \) exists it is called a **Satisfying Assignment**.
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\[
(x_1 \lor x_2 \lor x_3) \land (\neg x_1 \lor \neg x_2) \land (\neg x_2 \lor \neg x_3) \in SAT?
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**Def** $\phi(\vec{x}) \in \text{SAT}$ if there is $\vec{b}$ such that $\phi(\vec{b}) = T$. If $\vec{b}$ exists it is called a **Satisfying Assignment**.

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**Yes** $x_1 = T$, $x_2 = F$, $x_3 = F$. 

If $x_1 = T$ then $x_3 = T$, $x_2 = F$. NO GOOD.

If $x_2 = F$ then $x_2 = F$. NO GOOD.
**Satisfiability (SAT)**

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$$(x_1 \lor x_2 \lor x_3) \land (\neg x_1 \lor \neg x_2) \land (\neg x_2 \lor \neg x_3) \in \text{SAT?}$$

**Yes** $x_1 = T, x_2 = F, x_3 = F$.

$$(x_1 \lor \neg x_2) \land (\neg x_1 \lor x_3) \land (\neg x_2 \lor \neg x_3) \land x_2 \in \text{SAT?}$$

If $x_1 = T$ then $x_3 = T, x_2 = F$. NO GOOD.

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**Satisifiability (SAT)**

**Def** $\phi(\vec{x}) \in SAT$ if there is $\vec{b}$ such that $\phi(\vec{b}) = T$.
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$$(x_1 \lor x_2 \lor x_3) \land (\neg x_1 \lor \neg x_2) \land (\neg x_2 \lor \neg x_3) \in SAT?$$

**Yes** $x_1 = T$, $x_2 = F$, $x_3 = F$.

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Def $\phi(\vec{x}) \in \text{SAT}$ if there is $\vec{b}$ such that $\phi(\vec{b}) = T$. If $\vec{b}$ exists it is called a Satisfying Assignment.

$(x_1 \lor x_2 \lor x_3) \land (\neg x_1 \lor \neg x_2) \land (\neg x_2 \lor \neg x_3) \in \text{SAT}$?

Yes $x_1 = T$, $x_2 = F$, $x_3 = F$.

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Complexity of Satisfiability

**SAT Problem** Given $\phi$, determine if $\phi \in \text{SAT}$.

One Approach

Form Truth Table and see if any of the rows are T. This is often called a brute force search.

What are the PROs and CONS of this approach?

1. **PRO** Easy conceptually. Easy to code up.
2. **CON** Takes time roughly $2^n$ in the worst case.
3. **CAVEAT** Might do well on a formula that is in SAT since the algorithm can quit as soon as it finds a satisfying assignment.

On the next few slides discuss the following:

1. Is there a better algorithm?
2. Is there a class of formulas for which there is a better algorithm?
3. Is this problem interesting to people outside of Logic?
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- YES
- NO
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**YES** If $\phi$ is in 3-CNF form (we'll define that later) then there exists a randomized $1.306^n$ algorithm.
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YES and UNKNOWN TO SCIENCE

YES If $\phi$ is in 3-CNF form (we’ll define that later) then there exists a randomized $1.306^n$ algorithm.

UNKNOWN TO SCIENCE If there are no restrictions on the formula, then unknown if there is an algorithm better than $\sim 2^n$. 
What is Better?

There are many algorithms that work in time $\alpha^n$ for some $1 < \alpha < 2$. 

- These algorithms are very clever but are still Brute Force Search with Tricks.
- We want to say An Algorithm that is NOT brute force Search with Tricks. How can we define that?

Contrast:
- There is an algorithm for SAT that takes $\sim (1.1)^n$.
- There is an algorithm for SAT that takes $\sim n^{100}$.

In practice the $(1.1)^n$ algorithm is better. However, the $n^{100}$ algorithm is not doing brute force search!
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We now have our clean question:

**Is SAT in Polynomial Time?**

**Question**

If SAT is in time $n^{100}$, why do we care?

**Answer**

If SAT is in time $n^{100}$, then there is an algorithm that solves SAT that is not doing brute force search. It is doing something clever. That cleverness can likely be used to come up with a much better algorithm.

**Notation**

We denote Polynomial Time by $P$. 
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**Notation** We denote Polynomial Time by $P$. 
Is There a Class of Formulas for Which SAT is in P?

We define several variants of SAT:

1. SAT is the set of all formulas that are satisfiable. That is, \( \phi(\vec{x}) \in \text{SAT} \) if there exists a vector \( \vec{b} \) such that \( \phi(\vec{b}) = T \).

2. CNFSAT is the set of all formulas in SAT of the form \( C_1 \land \cdots \land C_m \) where each \( C_i \) is an \( \lor \) of literals.

3. \( k \)-CNFSAT is the set of all formulas in SAT of the form \( C_1 \land \cdots \land C_m \) where each \( C_i \) is an \( \lor \) of exactly \( k \) literals.

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2-CNFSAT is $C_1 \land \cdots \land C_m$ where each $C_i$ is an $\lor$ of exactly 2 literals.
2-CNFSAT is in $P$

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$$(x \lor y).$$
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Usually called 2-SAT.
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Example $(x_1 \land \neg x_2 \land x_3) \lor \cdots$ means you can put anything you want there. Without knowing anything else, this formula is satisfiable.

Set $x_1 = T, x_2 = F, x_3 = T$.

More Generally Given $\phi = C_1 \lor \cdots \lor C_m$ where each $C_i$ is a $\land$ of literals,

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Is 3-CNFSAT in P?

Is 3-CNFSAT in P? Vote:

- YES, and this is known (though perhaps complicated). Maybe it uses Ramsey Theory so I will be teaching it in my other class.
- NO, and this is known, and the proof is difficult (proving things can’t be done is usually hard). Maybe it uses Ramsey Theory so I will be teaching it in my other class.
- UNKNOWN TO SCIENCE.

The (1.306) $n$ algorithm is the best algorithm we know.

What Lower Bounds Are Known

It is known that 3-CNFSAT cannot be done in $n^{1.8}$ time and log-space.

How Long Has It Been Open For?

First posed in 1971, though see next slide.
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Is 3-CNFSAT in P?

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▶ YES, and this is known (though perhaps complicated). Maybe it uses Ramsey Theory so I will be teaching it in my other class.

▶ NO, and this is known, and the proof is difficult (proving things can’t be done is usually hard). Maybe it uses Ramsey Theory so I will be teaching it in my other class.

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How Long Has It Been Open For? First posed in 1971, though see next slide.
Is this problem interesting?

Consider the following problems:

1. Traveling Salesperson Problem (TSP)
   Given $n$ cities and how much it costs to go from any city to an city, determine the cheapest way to visit all cities. Studied since the 1930's.

2. Scheduling
   Given $n$ rooms and when they are free, and given $m$ people who are requesting them for certain timeslots, can you accommodate all of them? Studied since the 1880's.

The following is known:

- (3-SAT is in P) $\leftrightarrow$ (TSP is in P) $\leftrightarrow$ (SCHED is in P).

There are thousands of problems equivalent to SAT. Hence:

- The complexity of 3-SAT is important since it relates to the complexity of many other problems.
- Many of the problems 3-SAT is equivalent to have been worked on for 90 or more years; hence, it is unlikely they are in P. Hence it is unlikely that 3-SAT is in P.
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Proper Terminology and What Do People In the Know Think?

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More generally, if you now a problem is equivalent to SAT then you know that you should not look for an optimal poly time solutions. There are many other options to try.
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Its not all Bad News II

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He then ran out of room; however, his grandmother (my wife’s sister) tells me he can go all the way to 2048.