Honors Homework 1

Morally Due Mon Feb 12 at 10:00AM

1. (0 points) What is your name? Write it clearly.

2. (40 points)

   (a) (10 points) You go to a room with 3 people A_1, A_2, A_3. 1 is normal and 2 are truth tellers. Ask YES-NO questions to them to try to determine who is who. Try to make the number of questions as small as possible. (Questions are sequential: Ask a question to A_1, and based on the answer decide who to ask what.)

   (b) (10 points) You go to a room with 4 people A_1, A_2, A_3, A_4. 1 is normal and 3 are truth tellers. Ask YES-NO questions to them to try to determine who is who. Try to make the number of questions as small as possible. (Questions are sequential: Ask a question to A_1, and based on the answer decide who to ask what.)

   (c) (20 points) You go to a room with n people $A_1, \ldots, A_n$. 1 is normal and $n-1$ are truth tellers. Ask YES-NO questions to them to try to determine who is who. Try to make the number of questions as small as possible. (Questions are sequential: Ask a question to A_1, and based on the answer decide who to ask what.)

3. (30 points. This is NOT really a math problem) For this problem we use the following definitions of $\wedge$, $\vee$, $\neg$ and are using them on variables with values in $[0, 1]$.

   - $x \wedge y = xy$ (Multiplication)
   - $x \vee y = x + y - xy$
   - $\neg x = 1 - x$.

   Give two sentences A and B in English such that (a) you would give both of them a truth value of 0.9. (b) You would give $A \wedge B$ a truth value BIGGER than the 0.81 that the rules give. (This problem shows that the rules above might not correspond to our intuitions.)

4. (30 points) For this problem we use the following definitions of $\wedge$, $\vee$, $\neg$ and are using them on variables with values in $[0, 1]$.

   - $x \wedge y = \min\{x, y\}$
   - $x \vee y = \max\{x, y\}$
   - $\neg x = 1 - x$.

   (a) Write $x \implies y$ in terms of min, max, and arithmetic operations.