Honors Homework 3
Morally Due Mon April 1 at 10:00 AM.

Prove using **Structural Induction**:

1. (30 points) Define a set of strings S by

    - $\epsilon \in S$
    - If $\sigma \in S$, then
        - $b\sigma \in S$
        - $\sigma b \in S$
        - $\sigma aa \in S$
        - $aa\sigma \in S$

    Prove that every string in S contains an even number of a's.

2. (35 points) Let S be a set of ordered pairs of integers defined recursively by

    - $(0, 0) \in S$
    - If $(a, b) \in S$, then $(a + 2, b + 3) \in S$ and $(a + 3, b + 2) \in S$.

    Prove $5 | a + b$ when $(a, b) \in S$

3. (35 points) Prove by induction that a full binary tree has an odd number of nodes.