Homework 9

250H Spr 2024

Consider the BEE sequence. Try to prove that there are an infinite number of n such that $a_n \equiv 0 \pmod 9$ by following the proof for mod 7. Show where the proof breaks down.

We will try to show:

If $a_m \equiv 0 \pmod 9$ then there exists $m' > m$ such that $a_{m'} \equiv 0 \pmod 9$.

Proof:

Assume $a_m \equiv 0 \pmod 9$. a_m is used when calculating both a_{2m} and a_{2m+1}.

$$a_{2m} = a_{2m-1} + a_m \equiv a_{2m-1} \pmod 9$$

$$a_{2m+1} = a_{2m} + a_m \equiv a_{2m-1} \pmod 9$$

So, $a_{2m-1} \equiv a_{2m} \equiv a_{2m+1} \pmod 9$.

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Let $a_{2m-1} \equiv a_{2m} \equiv a_{2m+1} \equiv r \pmod 9$

$a_{4m-2} = a_{4m-3} + a_{2m-1} \equiv a_{4m-3} + r \pmod 9$

$a_{4m-1} = a_{4m-2} + a_{2m-1} \equiv a_{4m-3} + 2r \pmod 9$

$a_{4m} = a_{4m-1} + a_{2m} \equiv a_{4m-3} + 3r \pmod 9$

$a_{4m+1} = a_{4m} + a_{2m} \equiv a_{4m-3} + 4r \pmod 9$

$a_{4m+2} = a_{4m+1} + a_{2m+1} \equiv a_{4m-3} + 5r \pmod 9$

$a_{4m+3} = a_{4m+2} + a_{2m+1} \equiv a_{4m-3} + 6r \pmod 9$

$a_{4m+4} = a_{4m+3} + a_{2m+2}$

But we don't know anything about a_{2m+2}

Consider the BEE sequence. Try to prove that there are an infinite number of n such that $a_n \equiv 0 \pmod 9$ by following the proof for mod 7. Show where the proof breaks down.

$a_{4m+3} + r, \ldots, a_{4m+3} + 6r$ is $\equiv 0 \pmod 9$.

We can say $a_{4m+3} \equiv 1$ and $r \equiv 1$.

Therefore we get $\{a_{4m+3} + r, \ldots, a_{4m+3} + 6r\} \equiv \{2, 3, 4, 5, 6, 7\}$.

Hence we can never get 0.

Recall that $|SPS(1, . . . , n)| = [n(n+1)/2] + 1$. Assume n is large. What is $|SPS(1, . . . , n - 1, 2^n)|$?

We can have 2 kinds of sums:

- First consider sums that do not use 2^n .
  There are $|SPS(1, . . . , n - 1)|$ of those type of sums. So, we have $[(n-1)n/2] + 1$.
- Now consider sums that use 2^n.
  Since n is large all of the sums will be bigger than those in the first item.
  Therefore, they are DISJOINT from those in the first item.
  All of our sums will be of the form 2^n plus a sum in $SPS(1, . . . , n - 1)$.
  Hence there are $[(n-1)n/2] + 1$ of these sums.
  So we have $|SPS(1, . . . , n-1, 2 n)| = 2 [((n - 1)n/2) + 1] = (n-1)n+ 1 = n^2 - n + 2$.