

Homework 10

250H Spr 2024

Sandwich: Egg Salad OR Tuna Fish OR Cheese.

Fruit: Apple OR Orange OR Grapes OR Blueberries OR Strawberries.

Desert: Apple Sauce OR Cookie.

- How many ways can Bill make lunch for his Gnilrad?
 - $3 \times 5 \times 2 = 30$
- One day she complains: I don't want to have my Fruit be an apple, and my desert be Applesauce. Bill obeys her wishes. Now how many ways can Bill make lunch for her?
 - $30 - 3 = 27$
- One day she complains: I don't want to have my Fruit be an apple, and my desert be Applesauce AND I want 2 Sandwiches AND I want 3 Fruits. I still just want one Desert. Bill obeys her wishes. Now how many ways can Bill make lunch for her?
- $\binom{3}{2} \times \binom{5}{3} \times 2 - \binom{3}{2} = 3 \times 10 - 3 = 57$

What if the following hold

- Each A_i has x_1 elements.
- Each intersection of TWO sets has x_2 elements.
- Each intersection of THREE sets has x_3 elements.
- Each intersection of FOUR sets has x_4 elements.

Give an expression for $|A_1 \cup A_2 \cup A_3 \cup A_4|$ in terms of x_1, x_2, x_3, x_4 .

It should be much simpler than the general law.

$$4x_1 - 6x_2 + 4x_3 - x_4.$$

Let A_1, \dots, A_n be sets. Assume that, for $1 \leq i \leq n$, the intersection of i of these sets has size x_i . Give an expression for $|A_1 \cup \dots \cup A_n|$ in terms of x_1, \dots, x_n . You CANNOT use DOT DOT DOT. You can and should use a summation sign.

$$\sum_{i=1}^n (-1)^{i+1} \binom{n}{i} x_i$$

How many solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 = 100$$

with $x_1 \geq 1$, $x_2 \geq 2$, $x_3 \geq 3$, and $x_4 \geq 4$.

$$100 - 1 - 2 - 3 - 4 = 90 \text{ stars}$$

We have 3 bars, so

$$\binom{90 + 4 - 1}{4 - 1} = \binom{93}{3}$$