The Table Problem

True Story

The following people had dinner together in a restaurant:

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The following people had dinner together in a restaurant:

1. Bill and his Darling.

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1. Bill and his Darling.
2. Peggy and her husband Ted.

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1. Bill and his Darling.
2. Peggy and her husband Ted.
3. Jane and her husband Jon.

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How did they sit?

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The following people had dinner together in a restaurant:

1. Bill and his Darling.
2. Peggy and her husband Ted.
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How did they sit?

1. Bill & Darling **ACROSS**. On the LEFT END of the table.

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The following people had dinner together in a restaurant:

1. Bill and his Darling.
2. Peggy and her husband Ted.
3. Jane and her husband Jon.

How did they sit?

1. Bill & Darling **ACROSS**. On the LEFT END of the table.
2. Peg & Ted **NEXT**. Ted on her right; Bill on her right.

True Story

The following people had dinner together in a restaurant:

1. Bill and his Darling.
2. Peggy and her husband Ted.
3. Jane and her husband Jon.

How did they sit?

1. Bill & Darling **ACROSS**. On the LEFT END of the table.
2. Peg & Ted **NEXT**. Ted on her right; Bill on her right.
3. Jane & Jon **NEXT**. Jon on her left; Jon **NEXT** to Darling.

True Story

The following people had dinner together in a restaurant:

1. Bill and his Darling.
2. Peggy and her husband Ted.
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How did they sit?

1. Bill & Darling **ACROSS**. On the LEFT END of the table.
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Draw on the board.

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Work on this in groups.

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Answer on the **NEXT** slide.

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2. How many ways can you assign n couples to n tiles?

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Waitress: Answer is $A(n)n!2^n$ and
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She was a Math PhD students who was waitressing to pick up some extra cash.

Lets Figure out $A(n)$

After her break the waitress showed me the following

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Can we get $A(n)$ in terms of prior A's?

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Case 1 Left end has one couple, **ACROSS** from each other.

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Can we get $A(n)$ in terms of prior A's? Yes.

Case 1 Left end has one couple, **ACROSS** from each other. The rest of the tiling can be done $A(n-1)$ ways.

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Case 1 Left end has one couple, **ACROSS** from each other. The rest of the tiling can be done $A(n-1)$ ways.

Case 2 Left end has two couples, both are **NEXT** to each other.

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$A(1) = 1$

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$A(n) = A(n-1) + A(n-2)$

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$A(1) = 1$

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So $A(n) = F(n)$, the Fib Numbers!

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Peg and Jane said **Is that a lot?**

Darling said **Uh... Yes.**