Induction Problems

250H

Prove that a $2^n \times 2^n$ chess board with any one square removed can always be covered by L shaped tiles

Proof by Induction:

Base Case: Let $n = 0$. So, we have a single square chessboard. If we remove one square then the board is empty. Hence, it is also covered and our base case holds.

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Inductive Hypothesis: Assume for some $n \geq 1$, we can tile a 2^{n-1} x 2^{n-1} chessboard with any square removed.

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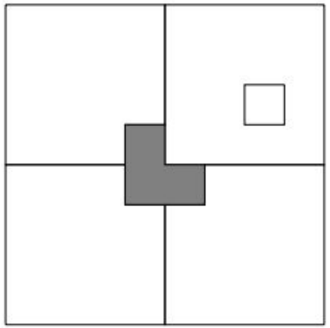
Inductive Step: Consider a 2^n x 2^n chessboard with a missing square. Divide the board into four quarters. Place a L tile in the center so that each quarter is missing a square.

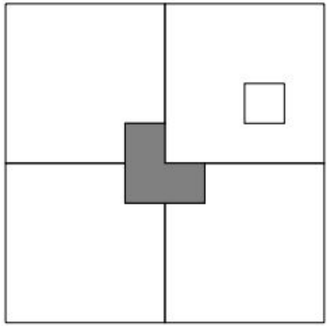


Prove that a 2^n x 2^n chess board with any one square removed can always be covered by L shaped tiles

Inductive Hypothesis: Assume for some n ≥ 1, we can tile a 2^{n-1} x 2^{n-1} chessboard with any square removed.

Inductive Step: Consider a 2^n x 2^n chessboard with a missing square. Divide the board into four quarters. Place a L tile in the center so that each quarter is missing a square.

By our inductive hypothesis, each of the quarters can be tiled, which gives us a way to tile a 2^n x 2^n chessboard. Hence, by PMI, we can tile a 2^n x 2^n chessboard.



More Induction Problems

1. $1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{1}{6}n(n+1)(2n+1)$
2. $1^5 + 2^5 + 3^5 + \cdots + n^5 = \frac{1}{12}n^2(n+1)^2(2n^2+2n-1)$
3. Prove that $5^{2n+1} + 2^{2n+1}$ is divisable by 7 for all $n \geq 0$
4. Prove that $2^n + 1$ is divisible by 3 for all odd naturals n