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Find Domains such that . . .

Consider:

$$(\exists x)(\forall y \neq x)[y > x]$$

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Give a domain where this is F. $\mathbb{Z}$ since, $\forall x$, $x - 1 < x$.

Expressing Math With Quantifiers

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I want to say x is even. How to do that with quantifiers.

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$$\mathrm{ONEFIVE}(x) \equiv (\exists y)[x = 5y + 1]$$

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NAH, we want -7 to be a prime.

PRIMES over $\mathbb{Z}$ (cont)

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$7 = i \times -i \times 7$.

We don't really want to count the i and $-i$.

Units

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Def Let D be a domain. The **units of D** are the elements of D that have a multiplicative inverse.

The Unit are the exceptions. If $x \in D$, u is a unit, and v is its inverse, then
$x = uvx$
We don't want to say x is not prime. u, v should not matter!

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Let D be any domain of numbers.
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$$(x \ne 0, x \notin \mathrm{UNIT}) \wedge (\forall y, z)[x = yz \rightarrow ((y \in \mathrm{UNIT}) \vee (z \in \mathrm{UNIT})].$$

So Thats why. . .

1) So thats why 1 is NOT a prime. In any domain D we have
Units, Primes, Composites, 0

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Primes in ONEFOUR

Elements of $\mathrm{ONEFOUR}$: $1, 5, 9, 13, 17, 21, 25$. We stop here.

1: a unit

5: a prime

9: a prime! Note that $3 \notin \mathrm{ONEFOUR}$ so cannot say $9 = 3 \times 3$.

13,17: Primes

21: a prime!

25: 5×5 are first composite.

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$$(\forall x)(\exists x_1, x_2, x_3, x_4)[x = x_1^2 + x_2^2 + x_3^2 + x_4^2]$$

Expressing Statements: Goldbach's Conjecture

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$$(\exists x)(\forall y > x)$$

$$[\mathrm{EVEN}(y) \rightarrow (\exists y_1, y_2)[\mathrm{PRIME}(y_1) \wedge \mathrm{PRIME}(y_2) \wedge (y = y_1 + y_2)]]$$

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$$[ODD(y) \rightarrow$$

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Note that using $\neg(\exists x, y) \equiv (\forall x, y)\neg$ ended up not having a $\neg$ in the final expression.

Order Notation

Sometimes We Don't Care About Constants

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EMILY: There are constants c, d, e such that my algorithm works in time $\leq cn^2 + dn + e$. OH, the algorithm only has this runtime when the number of variables is ≥ 100.

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BILL: What are c, d, e?
EMILY: Who freakin cares! I solved SAT without using brute force and you are concerned with the constants!

When Do/Don't We Care About Constants?

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If roughly n^2 then can we get it to roughly $n \log n$ or n?
Once we have exhausted all of our tricks to get it into (say) roughly n^2 time we THEN would do things to get the constant down, perhaps non-rigorous things.

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We leave it to the reader to prove that

$$18n^3 + 8n^2 + 12n + 1000 = O(n^3)$$

by finding the values of n_0, c, d.

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You will see $O()$ a lot in CMSC 351 and 451 when you deal with algorithms and want to bound the run time, roughly.

Other Ways to Use $O()$

$f \in n^{O(1)}$ means poly time.

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$f \in 2^{O(n)}$ means 2^{cn} for some c, and after some n_0.

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BILL: Who freakin cares! I showed SAT is not in poly time you are concerned with the constants!

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This notation is used to express that an algorithm **requires** some amount of time.

If I proved ...

If I proved that SAT requires $\Omega(n^3)$ time would I have solved P vs NP?

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