Rev For Mid I: Logic

Rev of Propositional Logic

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   Have class do some examples of this.
   2.3 A formula with n variables has a TT with 2^n rows.

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Discuss how you write a circuit tha tells, given an 8-bit number, outputs Y if prime and N if not prime.

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2. Can do a circiuit that inputs 3 bits and outputs the sum and the carry. Called a Full-Adder (HA).
3. Can combine HA and FAs to get create a circuit that adds two n-bit numbers.

Logical Equivalance

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2. Some well known equivalences:
   - 2.1 $\neg\neg p \equiv p$
   - 2.2 $\neg(p \vee q) \equiv \neg p \wedge \neg q$.
   - 2.3 $\neg(p \wedge q) \equiv \neg p \vee \neg q$.
   - 2.4 There are others.

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In any SAT assignment need $x_1 = T$ and $x_3 = F$ so $\neg x_1 \vee x_3$ is F.
Hence NOT in SAT.

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There are **thousands** of problems are equiv to SAT. Hence:

- The complexity of 3-SAT is **important** since it relates to the complexity of many other problems.
- Many of the problems 3-SAT is equivalent to have been worked on for 90 or more years; hence, it is unlikely they are in P. Hence it is unlikely that 3-SAT is in P.

Rev of Quantifier Logic

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$$\neg(\exists x)[P(x)]$$

$$\equiv$$

$$(\forall x)[\neg P(x)]$$

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The point of the math-with-quant slides is that we can STATE math of interest clearly using quantifiers.

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If you want to show this is false then have to find an $x \in \mathbb{N}$ such that

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You can't! So the statement is true.

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The list of conditions is on the next slide.

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2) D is finite and dense and has ≥ 2 elements.
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5) D infinite, has min, has max, and is NOT dense.
DOES EXIST: $\{-1, -\frac{1}{2}, -\frac{1}{3}, \ldots\} \cup \{\ldots, \frac{1}{3}, \frac{1}{2}, 1\}$

Sets

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A, B, U are all sets. U is the universe we are in (e.g., $\mathbb{R}$).

1. $A \cup B = \{x : x \in A \vee x \in B\}$.
2. $A \cap B = \{x : x \in A \wedge x \in B\}$.
3. $\overline{A} = \{x \in U : x \notin\}$.

Note that $\overline{A}$ only makes sense if you have a Universe.

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   1.2 $A \cap \emptyset = \emptyset$.

Power Set

Definition Of A is a set then **the powerset of A**, denoted $P(A)$, is the set of all subsets of A.

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If $A = \{4, 6, 9\}$ then
$P(A)$ is

$$\{\emptyset, \{4\}, \{6\}, \{9\}, \{4, 6\}, \{4, 9\}, \{6, 9\}, \{4, 6, 9\}\}$$