Timed Midterm

250H Spr 2024

Show that, for all $n \geq 1$, there is a formula $\varphi(x_1, . . . , x_n)$ that has exactly n^2 satisfying assignments.

- We can simply write out a truth table and have the first n^2 entries be true and the rest be false. Then we can use the truth table to create a formula that would have exactly n^2 satisfying assignments

Give a function $f(n)$ so that the following statement is FALSE
For all $n \geq 1000$, there is a formula $\varphi(x_1, . . . , x_n)$ that has exactly $f(n)$ satisfying assignments.

- Consider $f(n) = 2^{n+1}$. A formula that has n variables can have at most 2^n satisfying assignments. Therefore, we cannot have 2^{n+1} of them.

Let A = {a, 3, bill}. Write down all elements of the powerset of A

- P(A) = {∅, {a}, {3}, {bill}, {a, 3}, {a, bill}, {3, bill}, {a, 3, bill}}

Is the following statement True or False: If the powerset of A has exactly 7 elements then A is infinite.

- True.
- Let A have n elements. Then P(A) has to have 2^n elements. 2^n can never equal 7. Therefore our statement is: If false then A is infinite. The truth value of whether A is infinite does not matter. Therefore, the statement is vacuously true.

Is the following statement True or False: If the powerset of A has exactly 8 elements then A is infinite.

- False.
- Let A = {1, 2, 3}. The powerset of A has 8 elements: {∅, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, {1, 2, 3}}. But A is finite, so our statement must be false.

Fill in the blanks and justify.

- $(x \equiv 0 \pmod{3}) \land (y \equiv 0 \pmod{5}) \Rightarrow xy \equiv 0 \pmod{BLANK_1}$
    - If $x \equiv 0 \pmod{3}$, then $x = 3k$ for some k
    - If $y \equiv 0 \pmod{5}$, then $y = 5j$ for some j
    - So, $xy = (3k)(5j) = 15(kj)$
    - Therefore by definition, $xy \equiv 0 \pmod{15}$
- $(x \equiv 0 \pmod{a}) \land (x \equiv 0 \pmod{b}) \Rightarrow xy \equiv 0 \pmod{BLANK_2}$
    - If $x \equiv 0 \pmod{a}$, then $x = ak$ for some k
    - If $y \equiv 0 \pmod{b}$, then $y = bj$ for some j
    - So, $xy = (ak)(bj) = ab(kj)$
    - Therefore by definition, $xy \equiv 0 \pmod{ab}$

Fill in the blanks and justify.

- $(x \equiv 0 \pmod{6}) \wedge (x \equiv 0 \pmod{15}) \Rightarrow x + y \equiv 0 \pmod{BLANK_3}$
  - If $x \equiv 0 \pmod{6}$, then $x = 6k$ for some k
  - If $y \equiv 0 \pmod{15}$, then $y = 15j$ for some j
  - So, $x + y = (6k) + (15j) = 3(2k + 5j)$
  - Therefore by definition, $x + y \equiv 0 \pmod{3}$
- $(x \equiv 0 \pmod{a}) \wedge (x \equiv 0 \pmod{b}) \Rightarrow x + y \equiv 0 \pmod{BLANK_4}$
  - If $x \equiv 0 \pmod{a}$, then $x = ak$ for some k
  - If $y \equiv 0 \pmod{b}$, then $y = bj$ for some j
  - So, $x + y = (ak) + (bj)$
  - In order for, $x + y \equiv 0 \pmod{BLANK_4}$, We need to have a common factor between a and b to make as the mod. Therefore for simplicity sake, $x + y \equiv 0 \pmod{\gcd(a,b)}$