Untimed Midterm

250H Spr 2024

For each of the following sentences say if they are TRUE or FALSE and JUSTIFY your answer.

Let I = R − Q, the set of irrationals, for this problem.

- $(\forall x, y \in I)[x < y \rightarrow (x+y)/2 \in I)]$
  - False.
  - Let $x = -|z|$ and $y = |z|$ for $z \in I$. Then $(x+y)/2 = (-|z| + |z|)/2 = 0/2 = 0$. Since $0 \in Q$ our statement is false.
- $(\forall x, y \in I)[x < y \rightarrow (\exists z \in I)[x < z < y]$
  - True.
  - Consider $d = y - x$ and an n such that $1/n < d$. Let $z = x + 1/n$. We have $x < z < y$. Now lets make sure z is irrational. Let z be rational. Then by def, $z = a/b$ where $a, b \in Z$ and $b \neq 0$. Then
    $z = x + 1/n$
    $a/b = x + 1/n$
    $x = a/b - 1/n$
    But that would mean x is rational. Hence z is irrational.

For each of the following sentences say if they are TRUE or FALSE and JUSTIFY your answer.

Let I = R − Q, the set of irrationals, for this problem.

- $(\forall x, y \in I)[x < y \rightarrow (\exists z \in Q)[x < z < y]$
  - True
  - Consider, $x = n_x + 0.x_1x_2x_3 \ldots$ where $n_x \in N$ and $y = n_y + 0.y_1y_2y_3 \ldots$ where $n_y \in N$.
    Let $n_x < n_y$ and let $z = n_x + 1$.
    Let i be such that $x_i < y_i$. Note if no i existed then x = y, so a i must exist
    Consider $z = n_x + 0.x_1x_2 \cdots (x_{i-1})(x_i + 1)$.
    This would give us $x < z < y$.
    We can see that $z \in Q$ as the decimal part would terminate at the x_i place. If we have a decimal that terminates, it can be turned into a fraction and thus $z \in Q$

For each of the following sentences say if they are TRUE or FALSE and JUSTIFY your answer.

Let I = R − Q, the set of irrationals, for this problem.

- $(\forall x, y \in Q)[x < y \rightarrow (\exists z \in I)[x < z < y]$
  - True
  - Let d = y-x. Consider an n such that $1/(n\sqrt{2}) < d$.
    Let $z = x + 1/(n\sqrt{2})$.
    Note z is irrational as $1/(n\sqrt{2})$ is irrational.
    We also have $x < z < y$.

Let $f(x_1, x_2, x_3, x_4, x_5) = 1$ if exactly two inputs are 1 and $x_5 = 0$. Else 0.

- How many rows are in the Truth Table for f?
  - $2^5 = 32$
- How many rows of the truth table have 1 output?
  - 6
- Write down all of the rows of the Truth Table that output 1.
  - (T, T, F, F, F)
    (T, F, T, F, F)
    (T, F, F, T, F)
    (F, T, T, F, F)
    (F, T, F, T, F)
    (F, F, T, T, F)
- Write a DNF formula for f using the partial truth table in the last part.
  - $(\neg x_1 \land \neg x_2 \land x_3 \land x_4 \land \neg x_5) \lor (\neg x_1 \land x_2 \land \neg x_3 \land x_4 \land \neg x_5) \lor (\neg x_1 \land x_2 \land x_3 \land \neg x_4 \land \neg x_5) \lor$
    $(x_1 \land \neg x_2 \land \neg x_3 \land x_4 \land \neg x_5) \lor (x_1 \land \neg x_2 \land x_3 \land \neg x_4 \land \neg x_5) \lor (x_1 \land x_2 \land \neg x_3 \land \neg x_4$

Let $f(x_1, x_2, x_3, x_4, x_5) = 1$ if exactly two inputs are 1 and $x_5 = 0$. Else 0.

- You used a trick to avoid writing down that TT. Name a function $g(x_1, \ldots, x_n)$ where the trick would save you LOTS of time.
    - $g(x_1, \ldots, x_n) = 1$ iff only one row results in true
    - Instead of writing out a truth table of size 2^n you can just write down the single row that is true.
- Name a function $h(x_1, \ldots, x_n)$ where the trick would NOT save LOTS of time. Explain why.
    - $h(x_1, \ldots, x_n) = 1$ iff all rows result in true
    - All rows result in true, so all 2^n rows would need to be written down.