Homework 09, MORALLY Due April 28

1. (0 points) What is your name.

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2. (10 points)

   (a) (10 points) Show that

   $$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

   with a combinatorial proof.

   (Hint: Show that the RHS solves the question of how many ways to choose k objects out of n objects.)

   (b) (0 points but you'll need this for a later problem on this HW set and maybe later in life as well.)

   By convention $(\forall n \geq 0)[\binom{n}{0} = 1]$ and $(\forall k \geq 1)[\binom{0}{k} = 0]$.

   From Part 1 that $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$.

   Use these two equations to write a program that will do the following:

   Given N, K outputs $\binom{k}{n}$ for all $0 \leq k \leq K$ and $0 \leq n \leq N$.

   Run this program for $N = 52$ and $K = 6$.

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3. (30 points) Oliver-Poker is poker with a normal deck, but each player gets SIX cards.

   For all questions here answer it BOTH in terms of binomial coefficient (e.g.,

$$\frac{\binom{12}{8}}{\binom{52}{6}}$$

)

   and as an actual number to four places (e.g., 0.2192).

   (For that you will use the output of the program you wrote in Problem 2.)

   (a) (10 points) What is the probability of getting three 2-of-a-kinds?

   Example: $(2\heartsuit, 2\spadesuit, 4\heartsuit, 4\diamondsuit, 8\spadesuit, 8\clubsuit)$ is three 2-of-a-kind.

   Counterexample: $(2\heartsuit, 2\spadesuit, 2\clubsuit, 2\diamondsuit, 8\spadesuit, 8\clubsuit)$ DOES NOT count as three 2-of-a-kind.

   (b) (10 points) What is the probability of getting a 2-of-a-kind and a 4-of-a-kind?

   Example: $(2\heartsuit, 2{,}2\heartsuit, 2\diamondsuit, 8{,}8\clubsuit)$ is a 4-of-a-kind and a 2-of-a-kind.

   (c) (10 points) What is the probability of getting a two 3-of-a-kind?

   Example: $(2\heartsuit, 2\spadesuit, 2\heartsuit, 8\diamondsuit, 8\spadesuit, 8\clubsuit)$ is two 3-of-a-kind.

   (d) (0 points) Once its approved Oliver (also called *Poptart*) will be teaching a STIC on Poker (real poker, not this problem). Consider taking it!!

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4. (30 points) Show that there is no way to load two 8-sided dice to get fair sums.

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5. (30 points)

(a) (0 points but you have to do it) Bill throws 2 fair 8-sided dice that are labeled in the normal way (each dice has faces with 1,2,3,4,5,6,7,8). For each $0 \le 2 \le 16$, give the probability of the sum of the dice being i.

(b) (30 points) Give two 8-sided dice that are NOT labelled $(1, 2, 3, 4, 5, 6, 7, 8)$. which, when rolled, give the same probabilities from Part a. (The labels have to all be natural numbers that are ≥ 1.)

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Let one dice be labelled $(a_1, \ldots, a_8)$ and the other one be labelled $(b_1, \ldots, b_8)$. Hence the generating function for the number-of-ways-to-get-number i is

$$(x^{a_1} + \cdots + x^{a_8})(x^{b_1} + \cdots + x^{b_8})$$

We set this equal to the gen function for two 8-sided dice

$$(x^{a_1} + \cdots + x^{a_8})(x^{b_1} + \cdots + x^{b_8}) = (x^8 + \cdots + x^1)^2$$

So we need to factor $(x^8 + \cdots + x^1)^2$:

$(x^8 + \cdots + x^1)^2 = (x^4 + 1)^2(x^2 + 1)^2(x + 1)^2 x^2$.

We need each factor to have the sum of coefficients equals 8.

We need each factor to have 0 constant term.

We need the two factors to be different, else we will likely get standard dice.

There are TWO possibilities:

1) $(x^5 + 2x^4 + 2x^3 + 2x^2 + x)(x^{11} + x^9 + 2x^7 + 2x^5 + x^3 + x)$

This yields 8-sided dice with the following labels:

$(5, 4, 4, 3, 3, 2, 2, 1)$ and $(11, 9, 7, 7, 5, 5, 2, 1)$

2) $(x^7 + 2x^6 + x^5 + x^3 + 2x^2 + x)(x^9 + 2x^7 + 2x^5 + 2x^3 + x)$

This yields 8-sided dice with the following labels:

$(7, 6, 6, 5, 3, 2, 2, 1)$ and $(9, 7, 7, 5, 5, 3, 3, 1)$

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