HW 8 CMSC 389. DUE Jan 16
SOLUTIONS

WARNING- THIS HW IS THREE PAGES

NOTE- THIS HW ALSO HAS WAYS TO GET MORE POINTS ON MIDTERM TWO.

NOTE- for this HW you should use CHOOSE or factorials or powers in your answer if appropriate. For example, if the answer is $\binom{9}{3}$ then just leave it in that form and DO NOT make it into an actual number. If the answer is naturally expressed as 8×7 then leave it in that form and DO NOT tell me that its 56.)

1. (0 points) What is your name? Write it clearly. Staple your HW. When is the final? Are you free then? (if not then SEE ME IMMEDIATELY)

2. (60 points) Bill makes his Darling lunch everyday. He gives her ONE Sandwich (Peanut butter and Jelly OR tuna fish OR egg salad OR tomato and cheese), ONE fruit (Apple OR Orange OR Pear OR Mango OR Grapes OR strawberries), and ONE snack (cookie, cake, applesauce).

   (a) How many DIFFERENT lunches can Bill make his darling?

   (b) Assume that Darling DOES NOT like to have an Apple AND Applesauce. Now how many ways DIFFERENT lunches can Bill make his darling?

   (c) Darling is HUNGRY! She wants THREE sandwitches, FOUR fruits, and TWO Snacks. NOW how many ways DIFFERENT lunches can Bill make his darling? (It's OKAY to have an apple and applesauce.)

   (d) Darling is HUNGRY! She wants THREE sandwiches, FOUR fruits, and TWO Snacks. But she DOES NOT want an apple and applesauce. NOW how many ways DIFFERENT lunches can Bill make his darling?

   (e) Darling says SURPRISE ME! give me SOME nonzero number of sandwiches, SOME nonzero number of fruits, SOME nonzero number of snacks. NOW how many ways DIFFERENT lunches can Bill make his darling? (It's OKAY to have an apple and applesauce.)

(f) Darling says SURPRISE ME! give me SOME nonzero number of sandwiches, SOME nonzero number of fruits, SOME nonzero number of snacks. But DO NOT give me an apple AND applesauce. NOW how many ways DIFFERENT lunches can Bill make his darling?

SOLUTION TO PROBLEM 2

a) There are FOUR types of sandwitches, SIX types of fruit, THREE types of snack so the answer is $4 \times 6 \times 3$.

b) We do this TWO ways:

WAY ONE: How many lunches HAVE both Apple and Applesauce. Since that takes care of Fruit and snack, all thats left is sandwitch. So there are FOUR lunches that have BOTH. So the answer is $(4 \times 6 \times 3) - 4$.

WAY TWO: Lets break the problem into three parts:

PART ONE: NEITHER Apple NOR Applesauce: $4 \times 5 \times 2$.

PART TWO: APPLE but NO APPLESAUSE: $4 \times 1 \times 2$

PART THREE: APPLESAUCE but NOT APPLE: $4 \times 5 \times 1$

So the answer is

$$(4 \times 5 \times 2) + (4 \times 1 \times 2) + (4 \times 5 \times 1)$$

c) How many ways are there to CHOOSE 3 types of sandwitches out of 4. $\binom{4}{3}$. Similar for the other choices. So the answer is $\binom{4}{3}\binom{6}{4}\binom{3}{2}$.

d) We do this TWO ways:

WAY ONE: How many lunches HAVE both Apple and Applesauce. We still have $\binom{4}{3}$ ways to pick our SET OF sandwitches. However we want 4 fruits out of 6 and one is already decided, so we can pick our SET OF fruit $\binom{5}{3}$. Similar, our SET OF snacks is can be chosen $\binom{2}{1}$. SO the number of lunches that HAVE both an apple and applesauce is $\binom{4}{3}\binom{5}{3}\binom{2}{1}$. Hence the answer is

$$\binom{4}{3}\binom{6}{4}\binom{3}{2} - \binom{4}{3}\binom{5}{3}\binom{2}{1}.$$

WAY TWO: Lets break the problem into three parts:

PART ONE: NEITHER Apple NOR Applesauce: $\binom{4}{3} \times \binom{5}{4} \times \binom{2}{2}$

PART TWO: APPLE but NO APPLESAUSE: $\binom{4}{3} \times \binom{5}{3} \times \binom{2}{2}$

PART THREE: APPLESAUCE but NOT APPLE: $\binom{4}{3} \times \binom{5}{4} \times \binom{2}{1}$

So the answer is

$$\binom{4}{3} \times \binom{5}{4} \times \binom{2}{2} + \binom{4}{3} \times \binom{5}{3} \times \binom{2}{2} + \binom{4}{3} \times \binom{5}{4} \times \binom{2}{1}$$

e) Darling wants a NONEMPTY set of sandwitches. How many NONEMPTY sets are there of sandwitches? I would say 2^6-1. Others say, correctly, $\binom{6}{1}+\binom{6}{2}+\cdots+\binom{6}{6}$ but I won't use that (if you did you got full credit).

Cut to the chase: The answer is $(2^6-1)(2^4-1)(2^3-1)$.

f) We do this TWO ways:

WAY ONE: How many lunches HAVE both Apple and Applesauce. We still have (2^6-1) sets of sandwitches. For Fruit- we MUST take an apple so there are 3 fruits left and we can take ANY (including nonempty) subset of them, so that 2^3. For Snack- similarly 2^2 possible sets. SO

The answer is $(2^6-1)(2^4-1)(2^3-1)-(2^6-1)(2^3)(2^2)$.

WAY TWO: Lets break the problem into three parts:

PART ONE: NEITHER Apple NOR Applesauce: $(2^6-1)(2^3-1)(2^2-1)$

PART TWO: APPLE but NO APPLESAUSE: $(2^6-1)(2^3)(2^2-1)$

PART THREE: APPLESAUCE but NOT APPLE: $(2^6-1)(2^3-1)(2^2)$.

So the answer is the SUM of these three.

3. (40 points) Recall that a GRAPH is a set of vertices V and their are edges between some of them. Let K_n be the graph that has EVERY edge in it. This is called THE KOMPLETE GRAPH ON n VERTICES.

   (a) How many edges are in the complete graph on n vertices?

(b) How many ways are there to 2-COLOR the edges of K_n? For example, the edges of K_4 are $12, 13, 14, 23, 24, 34$ and we could just do

12 RED

13 RED

14 BLUE

23 BLUE

24 BLUE

34 RED

(There are NO restrictions on the coloring.)

(c) How many ways are there to 3-COLOR the edges of K_n?

(d) Find a 2-coloring of the EDGES of K_5 such that there are NO triangles of the same color. (For example, you can't have 12 and 24 and 41 ALL be RED.)

SOLUTION TO PROBLEM 3

a) We are just choosing 2 vertices out of n so this is $\binom{n}{2}$.

b) Think of the edges as being listed out. The FIRST one we color RED or BLUE. TWO choices The SECOND one we color RED or BLUE. TWO choices. Etc. so the answer is $2^{\binom{n}{2}}$.

c) Think of the edges as being listed out. The FIRST one we color RED or BLUE or GREEN. THREE choices The SECOND one we color RED or BLUE or GREEN. TWO choices. Etc. so the answer is $3^{\binom{n}{2}}$.

THINK ABOUT: what if we had 4 colors?

d) Omitted.

4. (0 points but please THINK ABOUT) Is there a 2-coloring of the EDGES of K_6 with NO triangles of the same color?

REDO Parts of the Midterm

1. (If you got 5 points off for the $e = 0$ case on Problem 2a on the midterm then DO this problem to get those 5 points!)

   Alice and Bob are going to do Secret Sharing with Cards. $n = 6$. Alice has $\{1, 2, 6\}$, Bob has $\{3, 4, 5\}$ and

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1. (If you got 5 points off for the $e = 0$ case on Problem 2a on the midterm then DO this problem to get those 5 points!)

   Alice and Bob are going to do Secret Sharing with Cards. $n = 6$. Alice has $\{1, 2, 6\}$, Bob has $\{3, 4, 5\}$ and Eve has NO CARDS.

   (a) How many secret bits can Alice and Bob share? (JUST GIVE A NUMBER)

(b) Describe how they would do this CAREFULLY.

(c) Give an EXAMPLE of how they would do this. Do the example IN FULL. If someone has to do something random make sure it looks somewhat random to me.

2. (If you got 5 points off on 5b then do this problem to get back those 5 points!) TRUE, FALSE, and WHY:

   Alice and Bob have set up Diffie-Helman so they can exchange 20 bits. They plan to use this to share four 5-bit numbers so that they can then use a 2×2 Matrix cipher. (NOTE- this is what they WANT TO DO. It is IRRELEVANT if a matrix cipher is a good idea.) Does this work?