HW 6, Due Jan 12

THIS HOMEWORK IS TWO PAGES LONG SO DO NOT MISS THE SECOND PAGE.

NOTE- The problems on this HW are NOT on material we covered BUT are not hard, or are stuff you should know from CMSC 250. Feel free to consult the web, though the web can be a blackhole. I am giving this HW so that I can build on this knowledge in Tuesday.

On ALL questions be COHERENT, CLEAR, CONCISE.

1. (40 points)

   (a) PROVE that for all a, b, c COMPLEX the equation $ax^2 + bx + c = 0$ has AT MOST two solutions. You can assume that all quadratic polynomials can be factored into linear terms.

   (b) Find ALL solutions to $x^2 + 7x + 12 \equiv 0 \pmod{20}$. You can do this by brute force. (There is a clever way which I will show you in class later.)

   (c) In the last question there were MORE THAN two solutions. Hence the proof you did in part b was CORRECT over the reals but DID NOT WORK over mod 20. Go through the proof carefully and state which step WORKS over the reals but DOES NOT WORK over mod 20.

   (d) Does there exists a, b, c, m, $a \neq 0$, such that $ax^2 + bx + c \equiv 0$ $\pmod{m}$ has more than 2016 solutions in the set $\{0, 1, \ldots, m\}$? If you think YES then give the polynomial. If you think NO then give the proof.

2. (20 points) Bill makes Darling's Lunch. It consists of a SANDWICH (either Turkey, Egg Salad, Tuna Fish, Tomato, or PBJ), FRUIT (apple, orange, grapefruit) and DESSERT (applesauce, candy bar).

   (a) How many lunches are there (e.g., (Turkey, orange, candy bar) is a lunch).

   (b) How many lunches are there if you CANNOT have both an apple and applesauce.

3. (40 points) In this problem you can use the notation ! for Factorial.

   (a) There are 20 people in the Zell family. When they get together EVERYONE hugs EVERYONE. How many hugs are there? (NOTE-People cannot hug themselves. If Vince and Luisa hug then thats ONE hug. Its not Vince hugs Luisa AND Luisa hugs Vince.) EXPLAIN your answer. If you do not explain you get ZERO points. Feel free to look on the web or a book to find out how to solve this sort of problem but then put your explanation IN YOUR OWN WORDS.

   (b) There are 30 people in the Xavier family. They all look different (that is, no twins, etc). We want to take a picture of all 30 of them. If they stand left to right (Vince, Luisa, Alice, Bob,...,Zelda) that will be a DIFFERENT picture than (Luisa, Alice, Bob,...,Vince). How many different pictures can there be? You must EXPLAIN your answer. Same rules as last part.

   (c) There are 30 people in the Xavier family. They all look different (that is, no twins, etc). We want to pick out 20 of them and We want to take a picture of all 20 of them. How many different pictures can there be? You must EXPLAIN your answer. Same rules as last part.

   (d) There are 30 people in the Xavier family. They all look different (that is, no twins, etc). How many ways are there to just choose 20 of them? Note that this differs from the last problem since we don't care how they are ordered. You must EXPLAIN your answer. Same rules as last part.