HW 7, Due Jan 13

1. (30 points)
   (a) Find ALL solutions to
   \[ x^2 + 15x + 5 \equiv 0 \pmod{21} \]
   (NOTE- the solutions are in \{0, 1, 2, \ldots, 20\}.
   (b) How would you do part 1 NOT using Brute force? COHERENT, CLEAR, CONCISE as usual.

2. (30 points) For this problem we are working mod 21.
   (a) Find a linear function \( f(x) = ax + b \) that has NO roots OR prove that there is no such linear function.
   (b) Find a linear function \( f(x) = ax + b \) that has EXACTLY one root OR prove that there is no such linear function.
   (c) Find a linear function \( f(x) = ax + b \) that has AT LEAST two roots OR prove that there is no such linear function.

3. (30 points) Let \( p \) be a prime such that \( p - 1 = 2q^2r \) where \( q, r \) are primes. Write a program that will, given \( p, q, r, g \), test of \( g \) is a generator. YOUR PROGRAM CAN USE THE POWER FUNCTION. So you can write things like if \( g^{q^4} \) is an odd Fibonacci number then

4. (10 points) Let \( p \) be a prime. How many affine ciphers are there mod \( p \)?