1. (0 points) READ the NOTES on SECRET SHARING

2. (30 points) Assume there is already a fast procedure to TEST if a number is prime. Call it TEST(n).
   
   (a) (15 points) Write pseudocode for an algorithm that, on input $N$, finds a SAFE prime between $N$ and $2N$ and SKIPS any number $n$ that is divisible by 2, 3, or 5.
   
   (b) (15 points) In your code for Part 1 you should have tested if $(n - 1)/2$ was prime to ensure that $n$ was a safe prime. You may have ended up testing numbers of this form that are divisible by 2, 3, or 5. SO lets make it faster: Write pseudocode for an algorithm that, on input $N$, finds a SAFE prime between $N$ and $2N$ and SKIPS at any number $n$ such that $n$ is divisible by 2, 3, or 5 OR such that $(n - 1)/2$ is divisible by 2, 3, or 5.

3. (OPTIONAL) Zelda wants to share a secret $s$ with $A_1, \ldots, A_{n+1}$ so that
   
   • $A_1$ and $A_2$ can determine the secret,
   • $A_2$ and $A_3$ can determine the secret,
   • $A_3$ and $A_4$ can determine the secret,
   • ...
   • $A_n$ and $A_{n+1}$ can determine the secret.

   Zelda uses the Random String Method.
   
   (a) Explain what Zelda does.
   
   (b) For any particular $i \in \{1, \ldots, n + 1\}$ how many random strings does $A_i$ get?

GO TO NEXT PAGE!
4. (OPTIONAL) Zelda has a secret $s$ in the integers mod 13 and she wants to give shares to $A_1, \ldots, A_{10}$ such that

- If $A_1, A_2$ and ANY three of $\{A_3, \ldots, A_{10}\}$ get together then they can find out the secret, but NO two can.
- Each person gets ONE string of length $s$.
- The scheme is information-theoretic secure.

Explain how Zelda can do this.

5. (40 points) Zelda has a secret and she wants to use the polynomial method over mod 17. She wants to share it with $A_1, \ldots, A_6$ such that if 4 of them get together they can find out the secret but if 3 of them get together they cannot. She wants to give everyone one share in $\{0, \ldots, 16\}$. Recall that she gives $A_i f(i)$. We present different scenarios.

(a) $A_1$ has 2, $A_2$ has 5, $A_3$ has 10. If they get together then can they determine the secret? If so then say how, if not then say why not. (HINT- this does NOT involve a lot of calculation.)

(b) $A_1$ has 2, $A_2$ has 5, $A_3$ has 10, $A_4$ has 0. If they get together then can they determine the secret? If so then say how, if not then say why not. (HINT- this does NOT involve a lot of calculation.)

(c) $A_1$ has 1, $A_2$ has 1, $A_3$ has 1, $A_4$ has 1. Has something gone wrong? Gee they all have the same number! If something has gone wrong then what is it. If not then determine the secret. (HINT- this does NOT involve a lot of calculation.)

(d) $A_1$ has 0, $A_2$ has 0, $A_3$ has 0, $A_4$ has 0. Has something gone wrong? Gee they all have the same number! If something has gone wrong then what is it. If not then determine the secret. (HINT- this does NOT involve a lot of calculation.)

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6. (30 points) Zelda has used polynomial secret sharing with $A_1, \ldots, A_9$ such that any two together can learn the secret, but one person alone cannot. She does this over mod 7. $A_1$ and $A_2$ get together! They plan to have $A_1$ reveal and then $A_2$ reveal.

$A_2$ is dishonest!

$A_1$ reveals his share and its 6. $A_2$ wants to lie and reveal a share so that $A_1$ thinks the secret is 3. Can he do this? If so then say what he reveals, and if not then show why not.