A CFL for $\Sigma^* - \{a^n b^n c^n : n \in \mathsf{N}\}$
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1 Introduction

$\Sigma^* - \{a^n b^n c^n : n \in \mathsf{N}\}$.

Note that this language is the union of the following languages

1. $\Sigma^* ba \Sigma^*$
2. $\Sigma^* ca \Sigma^*$
3. $\Sigma^* cb \Sigma^*$
4. $\{a^{n_1} b^{n_2} c^{n_3} : n_1 < n_2\}$.
5. $\{a^{n_1} b^{n_2} c^{n_3} : n_1 > n_2\}$.
6. $\{a^{n_1} b^{n_2} c^{n_3} : n_2 < n_3\}$.
7. $\{a^{n_1} b^{n_2} c^{n_3} : n_2 > n_3\}$.
8. $\{a^{n_1} b^{n_2} c^{n_3} : n_1 < n_3\}$.
9. $\{a^{n_1} b^{n_2} c^{n_3} : n_1 > n_3\}$.

We give a CFG for each of these and then use closure under union. It would be very easy to use the closure under union construction to get a real CFG for the union.

2 $\Sigma^* ba \Sigma^*$

We give a CFG for $\Sigma^* ba \Sigma^*$. The ones for
$\Sigma^* ca \Sigma^*$
$\Sigma^* cb \Sigma^*$
are similar.
$S \to TbaT$
$T \to aT$
$T \to bT$
$T \to cT$
$T \to e$.
Note that T can generate any element of Σ^*. Hence we get $\Sigma^* ba \Sigma^*$.

3 $\{a^{n_1}b^{n_2}c^{n_3} : n_1 < n_2\}$

We give CFG's for $\{a^{n_1}b^{n_2}c^{n_3} : n_1 < n_2\}$. The CFG's for
$\{a^{n_1}b^{n_2}c^{n_3} : n_1 > n_2\}$
$\{a^{n_1}b^{n_2}c^{n_3} : n_2 < n_3\}$
$\{a^{n_1}b^{n_2}c^{n_3} : n_2 > n_3\}$
are similar.
$S \to TBC$
$T \to aTb$
$T \to e$
$B \to Bb$
$B \to b$
$C \to cC$
$C \to e$
IDEA: T will generate a^nb^n. Then B will generate AT LEAST one b, so there will be more b's than a's. C will generate as any number of c's.

4 $\{a^{n_1}b^{n_2}c^{n_3} : n_1 < n_3\}$

We give CFG's for $\{a^{n_1}b^{n_2}c^{n_3} : n_1 < n_2\}$. The CFG's for
$\{a^{n_1}b^{n_2}c^{n_3} : n_1 > n_3\}$
is similar.
$S \to aTcC$
$T \to aTc$
$T \to B$
$B \to bB$
$B \to e$
$C \to cC$
$C \to c$.
IDEA: T will generate a^nTc^n. Then C will generate at least one more c. Then T will generate any number of b's.