HW 3 CMSC 452. Morally DUE Sep 30 (yes, Sept 30)
SOLUTIONS

1. (0 points) What is your name? Write it clearly. Staple your HW. When is the midterm? Where is the midterm? When is the final?

2. (20 points) Write a DFA for the language

$$\{(x, y) \mid x + 1 \equiv y \pmod 3\}$$

Circle the ACCEPT states. Box the REJECT WITH DIGNITY states. Do nothing to the stupid states.

How many states does your DFA have? How many are ACCEPT? How many are REJECT WITH DIGNITY? How many are stupid?

Note that all we care about is that $x - y \equiv 2 \pmod 3$.

3. (20 points) Write an DFA for the language

$$\{(x, y) \mid x + 1 \not\equiv y \pmod 3\}$$

How many states does your DFA have? How many are ACCEPT? How many are REJECT WITH DIGNITY? How many are stupid?

4. (20 points)

Using the DFA in problem 2 write an NDFA for the language

$$\{x \mid (\exists y)[x + 1 \not\equiv y \pmod 3]\}$$

5. (20 points) Let L_1 be regular via DFA $(Q_1, \Sigma, \delta_1, s_1, F_1)$. Let L_2 be regular via DFA $(Q_2, \Sigma, \delta_2, s_2, F_2)$. Assume $\$ \notin \Sigma$. Write the DFA for

$$L_1\$L_2 = \{x\$y \mid x \in L_1 \wedge y \in L_2\}.$$

SOLUTION

We create a DFA for $L_1\$L_2$.

$$(Q_1 \cup Q_2 \cup \{dump\}, \Sigma \cup \{\$\}, \delta, s_1, F_2)$$

where δ is defined as follows.

$\delta(q, \sigma) = \delta_1(q, \sigma)$ if $q \in Q_1$ and $\sigma \neq \$$

$\delta(q, \$) = s_1$ if $q \in F_1$

$\delta(q, \$) = dump$ if $q \notin F_1$

$\delta(q, \sigma) = \delta_2(q, \sigma)$ if $q \in Q_2$ and $\sigma \neq \$$

$\delta(q, \$) = dump$ if $q \in Q_2$.

6. (20 points) Let L_1 be regular via NDFA $(Q_1, \Sigma, \Delta_1, s_1, F_1)$. Let L_2 be regular via NDFA $(Q_2, \Sigma, \Delta_2, s_2, F_2)$. Write the NDFA for $L_1 L_2$ (NO, just a picture won't suffice- I want to see if you can do this rigorously.)

   SOLUTION

   The intuition is that the final states of M_1 goes via e-transition to the start state of M_2. This is true, and here is how to say it rigorously

   NDFA for $L_1 L_2$ is

$$(Q_1 \cup Q_2, \Sigma, \Delta, s_1, F_2)$$

   where Δ is defined as follows:

   If $q \in Q_1$, $\sigma \in \Sigma \cup \{e\}$ then $\Delta(q, \sigma) = \Delta_1(q, \sigma)$.

   If $q \in F_1$ then $\Delta(q, e) = \{s_2\}$.

   If $q \in Q_2$, $\sigma \in \Sigma \cup \{e\}$ then $\Delta(q, \sigma) = \Delta_2(q, \sigma)$.