HW 4 CMSC 452. Morally DUE Sep 30 (YES, the same day as HW 3) SOLUTIONS

1. (0 points) What is your name? Write it clearly. Staple your HW. When is the midterm?

2. (30 points) Write a Regular Expression for the languages A, B, C below. The alphabet is $\{a, b\}$.

   (a) $A = \{w \mid abab \text{ is a suffix of } w\}$

   (b) $B = \{w \mid$ the third to the last symbol of w is a b$\}$
   (Examples: $aaabaa$, $abaaaaabab$.)

3. (30 points) Consider the following alternative proof that if L is accepted by a DFA then L has a regular expression.

   L is accepted by DFA $(Q, \Sigma, \delta, s, F)$.

   Let $S(i, j, k)$ be the set of all string w such that $\delta(i, w) = j$ via a route that uses AT MOST k STATES AS INTERMEDIARIES

   (a) What is $S(i, j, 0)$.

   (b) Write $S(i, j, k)$ in terms of $S(i', j', k-1)$ in such a way that this can used to prove that if all $S(i', j', k-1)$ can be expressed as a regular expression then so can $S(i, j, k)$.

   SOLUTION

   $S(i, j, 0)$ uses ZERO states as intermediary so same as $R(i, j, 0)$:

   $S(i, j, 0) = \{\sigma \mid \delta(i, \sigma) = j\}$ if $i \neq j$.

   $S(i, i, 0) = \{\sigma \mid \delta(i, \sigma) = j\} \cup \{e\}$.

   NOW, we need to express $S(i, j, k)$ in terms of $S(i', j', k-1)$.

   Think of going from i to j using k states as going to JUST before k and then going to k.

$$S(i, j, k) = S(i, j, k-1) \cup \bigcup_{\sigma \in \Sigma} \bigcup_{\{L \mid \delta(L, \sigma) = j\}} S(i, L, k-1)\sigma$$

4. (40 points)

(a) Write an NDFA for the language

$$L_3 = \{w \mid \text{ the third to the last symbol of } w \text{ is a b}\}$$

(b) Use the NDFA to DFA converstion to write a DFA for L_3. How many states does it have.

(c) Let $n \in \mathsf{N}$. Write an NDFA for the language

$$L_n = \{w \mid \text{ the } n\text{-to the last symbol of } w \text{ is a b}\}$$

You may use DOT-DOT-DOT notation.

(d) If you were to do the NDFA to DFA converstion for the DFA for L_n then how many states would it have when minimized? Argue why this is true informally.

SOLUTION. And DFA for L_n requires 2^{n-1} states. We prove this!

Let M be a DFA for L_n.

Let $w_1, w_2, \ldots, w_{2^{n-1}}$ be all of the strings of length $n-1$. Note that $bw_i \in L_n$. Hence if we feed bw_i into M we get to a final state.

We show that, for all $i \neq j$, bw_i and bw_j end up in DIFFERENT states. Hence there are at least 2^{n-1} states.

Let bw_i goto state s and bw_j goto state t. We show that $s \neq t$. Assume, by way of contradiction that $s = t$. Let L be the first place where bw_i and bw_j differ. Assume that bw_i has a b in the Lth place, and bw_j has an a in the Lth place.

Note that $bw_i \in \{a,b\}^{L-1}b\{a,b\}^{n-L}$. Hence $bw_i a^L \in \{a,b\}^{L-1}b\{a,b\}^n \in L_n$.

Note that $bw_j \in \{a,b\}^{L-1}a\{a,b\}^{n-L}$. Hence $bw_i a^L \in \{a,b\}^{L-1}a\{a,b\}^n \notin L_n$.

If we run $bw_i a^L$ through M then we get to state s and then read a^L to end up in state r, which must be a final state.

If we run $bw_j a^L$ through M then we get to state $t = s$ and then read a^L to end up in that same state r.

Hence we have that the M accepts $bw_j a^L$ which it should not.

Thats just final states. What about non-final states? One can show that $b, bb, \ldots, b^n$ all lead to diffeent nonfinal states. Hence there are at least n non-final states.

Hence M has at least $2^{n-1} + n$ states.