CMSC 452 Final

1. This is a closed book exam, though ONE sheet of notes is allowed. You may NOT use Calculators. If you have a question during the exam, please raise your hand.

2. There are 5 problems which add up to 100 points. The exam is 2 hours.

3. In order to be eligible for as much partial credit as possible, show all of your work for each problem, write legibly, and clearly indicate your answers. Credit cannot be given for illegible answers.

4. After the last page there is paper for scratch work. If you need extra scratch paper after you have filled these areas up, please raise your hand.

5. Please write out the following statement: “I pledge on my honor that I will not give or receive any unauthorized assistance on this examination.”

6. Fill in the following:

   NAME :
   SIGNATURE :
   SID :

   SCORES ON PROBLEMS (FOR OUR USE)
   Prob 1: 
   Prob 2: 
   Prob 3: 
   Prob 4: 
   Prob 5: 
   TOTAL
1. (20 points) For this problem you may assume that $P \neq NP$ and that $NP \neq coNP$. For each of the following sets say if it is:

- REGULAR
- in P but NOT REGULAR
- in NP but not P
- DEC but not in NP
- $\Sigma_1$ but not DEC
- Not $\Sigma_1$.

No explanation needed BUT you get +4 if right, -2 if wrong. (Hint- DO NOT GUESS!!!!!!!!!!!!!!)

(a) $\{G \mid G$ is 2-colorable $\}$.

(b) $\{G \mid G$ has an independent set of size 12$\}$. 
   (NOTE: an independent set in a graph is a set of vertices such that there is NO edge between any of them)

(c) $\{a^n b^m a^n a^m \mid n, m \in N\}$.

(d) $\{(G, \rho) : \rho$ is a 3-coloring of $G$ $\}$.

(e) $\{a^n \mid n$ is NOT a square}$
2. (20 points) For this problem use the WS1S convention. Let $n \geq 10$. Draw a DFA for the following language (you will have to use DOT DOT DOT DOT notation).

$$\{(x, y) : x < y \text{ AND } x \equiv y \pmod{n}\}.$$ 

How many states does it have (as a function of $n$)?
3. (20 points) If $x$ is a string then $x^R$ is the string in reverse order. (Example: $aaba^R = abaa$.) If $L$ is a language then

$$L^R = \{ x \mid x^R \in L \}$$

(just take all strings in $L$ and reverse them.) Show that if $L \in NP$ then $L^R \in NP$. Use the existential definition of $NP$. 
4. (20 points). Let \((35, 47)\)-IP be the set of all pairs \(A, b\) such that the following is true

- \(A\) is a matrix of integers.
- \(b\) is a vector of integers.
- There is a vector \(x\) of integers from the set \(\{35, 47\}\) such that \(Ax \leq b\).
- The dimensions of \(A, b, x\) work out so that \(Ax \leq b\) makes sense.

Show that \(5-SAT \leq (35, 47)\)-IP.

(RECALL that 5-SAT is the set of all formulas of the form \(C_1 \land C_2 \land \cdots \land C_k\) such that each \(C_i\) is the OR of 5 literals.)
5. (20 points) Let $M_1, M_2, \ldots$, be a standard list of Turing Machines.

show that

$$\{ e : M_e \text{ halts on exactly 5 elements } \}$$

is in $\Sigma_2$.

(RECALL: $A \in \Sigma_2$ if there exists a decidable set $B$ such that

$$A = e : (\exists x)(\forall y)[(x, y, z) \in B].$$

Note that we can always merge quantifiers of the same type, so if you have something like

$$\{ e : (\exists x_1, x_2)(\forall y_1, y_2, y_3, \ldots, y_{10^{10}})[(x, y, z) \in B] \}$$

that’s fine for showing a set is in $\Sigma_2$. (If you really do have $10^{10}$ variables then. . . check your solution, its probably wrong.)
Scratch Paper