**CMSC 452 Final**

1. This is a closed book exam, though ONE sheet of notes is allowed. **You may NOT use Calculators**. If you have a question during the exam, please raise your hand.
2. There are 5 problems which add up to 100 points. The exam is 2 hours.
3. In order to be eligible for as much partial credit as possible, show all of your work for each problem, **write legibly**, and **clearly indicate** your answers. Credit **cannot** be given for illegible answers.
4. After the last page there is paper for scratch work. If you need extra scratch paper **after** you have filled these areas up, please raise your hand.
5. Please write out the following statement: "*I pledge on my honor that I will not give or receive any unauthorized assistance on this examination.*"

6. Fill in the following:

NAME :
SIGNATURE :
SID :

SCORES ON PROBLEMS (FOR OUR USE)

| |
|---|
| Prob 1: |
| Prob 2: |
| Prob 3: |
| Prob 4: |
| Prob 5: |
| TOTAL |

1. (20 points) For this problem you may assume that $P \neq NP$ and that $NP \neq coNP$. For each of the following sets say if it is:

   - REGULAR
   - in P but NOT REGULAR
   - in NP but not P
   - DEC but not in NP
   - $\Sigma_1$ but not DEC
   - Not $\Sigma_1$.

   No explanation needed BUT you get +4 if right, -2 if wrong. (Hint- DO NOT GUESS!!!!!!!!!!!!!!!)

   (a) $\{G \mid G$ is 2-colorable $\}$.

   (b) $\{G \mid G$ has an independent set of size 12$\}$.
   (NOTE- an independent set in a graph is a set of vertices such that there is NO edge between any of them)

   (c) $\{a^n b^m a^{n+m} \mid n, m \in N\}$.

   (d) $\{(G, \rho) : \rho$ is a 3-coloring of $G\}$.

   (e) $\{a^{n^2} \mid n$ is NOT a square$\}$

2. (20 points) For this problem use the WS1S convention. Let $n \geq 10$. Draw a DFA for the following language (you will have to use DOT DOT DOT notation).

$$\{(x, y) : x < y \text{ AND } x \equiv y \pmod{n}\}.$$

How many states does it have (as a function of $n$)?

3. (20 points) If $x$ is a string then $x^R$ is the string in reverse order. (Example: $aaba^R = abaa$.) If $L$ is a language then

$$L^R = \{x \mid x^R \in L\}$$

(just take all strings in $L$ and reverse them.) Show that if $L \in NP$ then $L^R \in NP$. Use the existential definition of $NP$.

4. (20 points). Let $(35, 47)$-IP be the set of all pairs $A, b$ such that the following is true

   - $A$ is a matrix of integers.
   - $b$ is a vector of integers.
   - There is a vector $x$ of integers from the set $\{35, 47\}$ such that $Ax \leq b$.
   - The dimensions of $A, b, x$ work out so that $Ax \leq b$ makes sense.

   Show that $5 - SAT \leq (35, 47)\text{-}IP$.

   (RECALL that 5-SAT is the set of all formulas of the form $C_1 \wedge C_2 \wedge \cdots \wedge C_k$ such that each $C_i$ is the OR of 5 literals.)

5. (20 points) Let $M_1, M_2, \ldots,$ be a standard list of Turing Machines. show that

$$\{e : M_e \text{ halts on exactly 5 elements } \}$$

is in $\Sigma_2$.

(RECALL: $A \in \Sigma_2$ if there exists a decidable set $B$ such that

$$A = e : (\exists x)(\forall y)[(x, y, z) \in B].$$

Note that we can always merge quantifiers of the same type, so if you have something like

$$\{e : (\exists x_1, x_2)(\forall y_1, y_2, y_3, \ldots, y_{10^{10}})[(x, y, z) \in B]\}$$

that's fine for showing a set is in $\Sigma_2$. (If you really do have $10^{10}$ variables then... check your solution, its probably wrong.)

Scratch Paper